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TECHNICAL MEMORANDUM NO. 60

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A FORTRAN IV PROGRAM TO COMPUTE THE INVERSE LAPLACE TRANSFORM AND PLOT THE RESPONSE OF A LINEAR SYSTEM SUBJECTED TO A FORCING FUNCTION

by

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Harold H. Burke

March 1970

U.S. ARMY ABERDEEN RESEARCH AND DEVELOPMENT CENTER
ARMY MATERIEL SYSTEMS ANALYSIS AGENCY
ABERDEEN PROVING GROUND, MARYLAND

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ARMY MATERIEL SYSTEMS ANALYSIS AGENCY

TECHNICAL MEMORANDUM NO. 60

MARCH 1970

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RDT&E Project No. 1P765801M1102

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JAAndrese/HHBurke/sjj
Aberdeen Proving Ground, Md.
March 1970

A FORTRAN IV PROGRAM TO COMPUTE THE INVERSE LAPLACE TRANSFORM AND PLOT THE RESPONSE OF A LINEAR SYSTEM SUBJECTED TO A FORCING FUNCTION

ABSTRACT

An existing program, which determines the inverse Laplace transform of a quotient of two polynomials, provides expanded systems analysis capabiltiy. The program complements a Root Locus Program (AMSAA Technical Memorandum No. 21) and a Frequency Response Program (AMSAA Technical Memorandum No. 69). The program uses a self-contained complex arithmetic routine and also a self-adjusting variable scale plotting technique. The plotting is done on a standard line printer and gives a time history plot of system response for a variety of input forcing functions.

A listing of the FORTRAN IV source deck and the corresponding flow chart of the program is shown in the appendixes. Also, several examples are given to introduce the user to the operating procedures and capabilities of the program.

TABLE OF CONTENTS

LIST OF FIGURES

LIST OF TABLES

LIST OF TABLES (Cont'd)

1. INTRODUCTION

One method of analyzing physical systems requires the use of Laplace transforms. The inverse Laplace transformation is necessary to analyze a system's time response to a forcing function.

Time constants, overshoots, resonant frequencies, and steady state errors are obtained by the time response of the system. By observing these characteristics, the designer is able to optimize the system response to a particular performance index.

Inverse Laplace transforms can easily be found in a handbook of Laplace transform pairs for the simpler control system equations. For more complex systems it is a tedious task to take the partial fractions representing the quotient, find the corresponding inverse Laplace transform of each individual term and sum them.

An existing FORTRAN IV program (References 1,2,3) has been adapted for system analysis application to relieve the computational burden. The program determines the inverse Laplace transform of a wide variety of high-order system equations. The transient response is graphically displayed for quick and easy analysis. The main features of the modified program are:

1. FORTRAN IV programming which needs no machine-oriented or object language.

2. Self-contained complex arithmetic (compatible with BRLESC).

3. Digital plotting with on-line printer.

---

[1] McCracken, Daniel D; FORTRAN With Engineering Applications; John Wiley and Sons, Inc., New York, 1967, pp. 196-214.

[2] Titus, C. K.; A General Card - Program for the Evaluation of the Inverse Laplace Transform; J. Assoc., Computing Machinery, 2, No. 1, January 1958.

[3] T. R. Boshkow,; Curve Plotting Routine for the Inverse Laplace Transform of Rational Functions; J. Assoc., Computing Machinery, 5, No. 1, January 1958.

4. Maximum order of numerator and denominator polynomials are 51 and 52 respectively.

5. Fast operating time.

Main features of the graphical display are:

1. Automatic scaling to give maximum viewing attenuation.

2. Preselected scaling to give uniformity.

3. Variable self-adjusting computing interval for quick running time.

4. Manual selection of up to 15 different real-time intervals for a particular function.

5. Automatic cut off at time when response is constrained within 1 X 10^{-K} for five consecutive computation intervals (where K is dependent upon the specific problem being solved).

Requirements of the Program

The quantities needed for problem solution are:

1. Coefficients of numerator and denominator of the system transfer function in polynomial form[4,5]

$$\frac{N(s)}{D(s)} = \frac{a_o s^n + a_1 s^{n-1} + a_2 s^{n-2} + \ldots a_{n-k} s^k}{b_o s^{m+n} + b_1 s^{m+n-1} + b_2 s^{m+n-2} + \ldots b_{m+n-\ell} s^\ell} \qquad (1)$$

[4] Burke, Harold, H. and Payne, Robert L.; A Linear Closed Loop System Analysis Procedure Using Line Printer Plots of Characteristic Equation Root Loci, AMSAA TM No. 21, November 1968, Army Materiel System Analysis Agency, Aberdeen Proving Ground, Maryland.

[5] Burke, Harold, H. and Payne, Robert L.; A Linear and Nonlinear Systems Analysis Tool: Line Printer Plots of Characteristic Equation Root Loci, Bode and Popov Plots of System Transfer Functions, AMSAA Technical Memorandum No. 69, Army Materiel Systems Analysis Agency, Aberdeen Proving Ground, Maryland. (In publication).

2. Roots and the multiplicity of the roots of the denominator.[4,5]

2. THEORY

The basic transfer function of a closed-loop system can be shown to be of the form given in Figure 1. For multiple-loop system, the G's and H's are readily expressed as sums of products of polynomials which are identified with the individual elements making up the complete system.

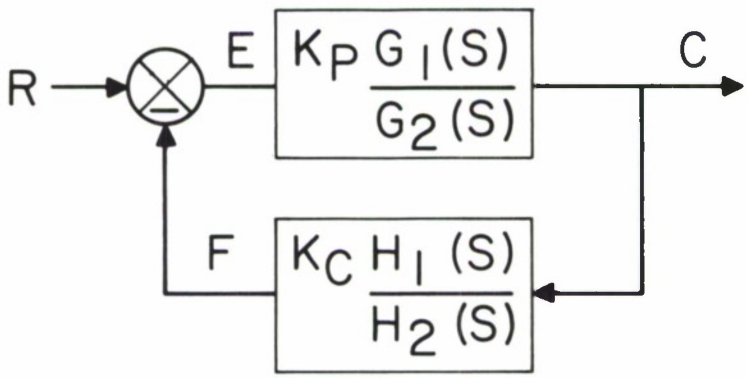

Figure 1. Linear Closed-Loop System

where
- R = System Input
- C = System Output
- F = System Feedback
- E = System Error
- K_p = Process Gain
- K_c = Controller Gain

The linear open-loop system transfer function is

$$\frac{F}{E} = \left[K_p \frac{G_1(s)}{G_2(s)}\right]\left[K_c \frac{H_1(s)}{H_2(s)}\right] . \tag{2}$$

---

[4] Ibid.
[5] Ibid.

The linear closed-loop system transfer function is

$$\frac{C}{R} = \frac{K_pG_1(s)H_2(s)}{K_cK_pG_1(s)H_1(s) + G_2(s)H_2(s)} \quad . \tag{3}$$

Equations (2) and (3) can be expressed as ratios of polynomials as shown in Equation (1). Rewritten in series form, Equation (1) becomes

$$\frac{N(s)}{D(s)} = \frac{\sum_{\mu=o}^{n-k} a_\mu s^{n-\mu}}{\sum_{i=o}^{m+n-\ell} b_i s^{m+n-i}} \quad , \tag{4}$$

where the numerator and denominator polynomials are different for the open- and closed-loop system transfer functions.

The factored form of the right-hand side of Equation (4) is

$$\frac{s^k \prod_{\mu=1}^{v} (s+z_\mu) \prod_{q=v+1}^{y} \left[s + (\sigma_q+j\omega_q)\right]\left[s + (\sigma_q-j\omega_q)\right]}{s^\ell \prod_{i=1}^{g} (s+p_i)^h \prod_{t=g+1}^{r} \left[s + (\sigma_t+j\omega_t)\right]\left[s + (\sigma_t-j\omega_t)\right]} \quad , \tag{5}$$

where $h = 1$ or 2

$$y = \frac{n - k - v}{2} \quad , \text{ and}$$

$$r = \frac{m + n - \ell - q}{2} \quad .$$

The inverse Laplace transform, c(t) for an arbitrary input forcing function r(t) of Equation (4) is

$$c(t) = \mathcal{L}^{-1}\left[c(s)\right] = \mathcal{L}^{-1}\; R(s)\,\frac{s^{k}\prod\limits_{\mu=1}^{v}(s+z_{\mu})\prod\limits_{q=v+1}^{y}\left[s+(\sigma_q+j\omega_q)\right]\left[s+(\sigma_q-j\omega_q)\right]}{s^{\ell}\prod\limits_{i=1}^{g}(s+p_i)^{h}\prod\limits_{t=g+1}^{r}\left[s+(\sigma_t+j\omega_t)\right]\left[(s+\sigma_t-j\omega_t)\right]}\,, \tag{6}$$

where R(s) is the Laplace transform of the input forcing function r(t). R(s) will be multiplied by the right-hand side of Equation (4) to obtain the Laplace transform given in Equation (6). The partial fraction expansion of Equation (6) is

$$c(t) = \mathcal{L}^{-1}\left[\sum_{j=\ell-k}^{1}\frac{K_j}{s^j} + \sum_{i=1}^{g}\frac{K_{ih}}{(s+p_i)^h} + \sum_{t=g+1}^{r}\frac{\vec{K}_t}{s^2+2\sigma_t\omega_t s+\omega_t^{\,2}}\right], \tag{7}$$

where K_j, K_{ih} and K_t are the residues at each denominator (pole) singularity of the right-hand side of Equation (6) and h = 1 or 2 .

The coefficients of non-repeated real roots are at origin

$$\underset{j=1}{K_j} = \underset{s=o}{\text{Limit}}\left[s\,\frac{C(s)}{R(s)}\right], \tag{8}$$

on real axis

$$\underset{h=1}{K_{ih}} = \underset{s=-p_i}{\text{Limit}}\left[(s+p_i)\,\frac{C(s)}{R(s)}\right]. \tag{9}$$

The coefficients of the partial fraction terms of Equation (7) related to the repeated real roots are

$$K_j = \left\{ \frac{1}{(j-1)!} \frac{d^{j-1}}{ds^{j-1}} \left[(s^{\ell-k}) \frac{C(s)}{R(s)} \right] \right\} \Bigg|_{s=o} , \tag{10}$$

and

$$K_{ih} = \left\{ \frac{1}{(h-1)!} \frac{d^{h-1}}{ds^{h-1}} \left[(s+p_i)^h \frac{C(s)}{R(s)} \right] \right\} \Bigg|_{s=-p_i} , \tag{11}$$

where h = 2.

The coefficients of the partial fraction terms of Equation (7) related to the complex roots are

$$\vec{K}_{t_1} = \underset{s = -(\sigma_t+j\omega_t)}{\text{limit}} \left\{ \frac{C(s)}{R(s)} [s + (\sigma_t+j\omega_t)] \right\} , \tag{12}$$

and

$$\vec{K}_{t_2} = \underset{s = -(\sigma_t-j\omega_t)}{\text{limit}} \left\{ \frac{C(s)}{R(s)} [s + (\sigma_t-j\omega_t)] \right\} , \tag{13}$$

where $\vec{K}_t$ in Equation (7) equals $\vec{K}_{t_1} + \vec{K}_{t_2}$.

Once these residues (K_j, K_{ih} and $\vec{K}_t$) have been determined, the time response of c(t) can be determined.

$$c(t) = \sum_{j=1}^{\ell-k} \frac{K_j t^{(j-1)}}{(j-1)!} + \sum_{i=1}^{g} \frac{K_{ih} t^{(h-1)}}{(h-1)!} e^{-p_i t} + \sum_{t=g+1}^{r} \vec{K}_{t_1} e^{-(\sigma_t+j\omega_t)}$$

$$+ \vec{K}_{t_2} e^{-(\sigma_t-j\omega_t)} , \qquad h = 1 \text{ or } 2 \tag{14}$$

or the transcendental form of Equation (14) is

$$c(t) = \sum_{j=1}^{\ell-k} \frac{K_j t^{(j-1)}}{(j-1)!} + \sum_{i=1}^{g} \frac{K_{ih} t^{(h-1)}}{(h-1)!} \quad e^{-p_i t} + \sum_{t=g+1}^{r} 2|K_t| e^{-\sigma t} \sin(\omega_t t + \angle K_t) \; .$$

$$h = 1 \text{ or } 2 \qquad (15)$$

3. GRAPHICAL METHODS

A plotting routine is used to graphically show the time response of the inverse Laplace transform, Equation (15). The routine varies the solution interval in accordance with the specific problem being solved. This adjustable time step is computed as a function of previous step size and the slope between consecutive points.

The variable time step is initiated by first finding the smallest time increment after the problem start time. This initial step is a function of the largest real root (fastest exponential rise) and of the largest imaginary root (highest frequency sine wave). The increment thereafter will be determined by the change of the slope between points. The expression for the derivative, $\dot{f}(t)$, is found by explicit differentiation of the function. The time step is then determined by means of comparison between the slope $\phi_i = \tan^{-1} \dot{f}(t)$ evaluated at t_i and the slope $\phi_i + 1$ evaluated at $t_i + \Delta t_i$. The solution time is left to the discretion of the programmer. Critical areas may be displayed by isolating the particular region of interest.

There will be a priority cutoff time if the response is constant within a difference of 1×10^{-K} for five consecutive computing intervals. (K is equal to 8 in the current program.) The response will then be assumed constant out to the stop time requested in the input data. This procedure is employed to allow for maximum sensitivity of the line printer plotting display.

It is not necessary to specify the upper and lower limits of the c(t) axis. The program will select a scale which permits the graph to be displayed with

maximum sensitivity. If no limits are specified, the program will adjust the c(t) axis scale dependent upon the maximum and minimum values of the response function contained in the computation interval.

Parametric plots may be directly compared by using the same c(t) amplitude scales. Only the values contained in the region bounded by the specified limits of the c(t) axis will be plotted.

4. DATA FORMATS

4.1 Inverse Laplace Transform.

1. General Description.

a. Computes the inverse Laplace transform parameters of the first, second and third order poles at the origin. No input data are necessary for this process because it is evaluated automatically.

b. Computes the inverse Laplace transforms of first- and second-order real poles not at the origin.

c. Computes the inverse Laplace transforms of the first-order complex poles.

2. Input.

Description	Columns	Data
(a) Coefficients of Numerator Polynomial	1-5 (Integer)	Order of term of polynomial (up to 51st order)
	6-20 (Floating Pt.)	Coefficients of corresponding term
(b) Coefficients of Denominator Polynomial	1-5 (Integer)	Order (up to 52nd)
	6-20 (Floating Pt.)	Coefficients of corresponding term

Description		Column	Data
(c)	Roots of Denominator	1-15 (Floating Pt.)	Real Part (σ_t or p_i)
	Poles of forms ($\sigma_t \pm j\omega_t$) and (p_i)	(16-30) (Floating Pt.)	Imaginary Part (ω_t)
		(31-45) (Integer)	Multiplicity of Root

3. Output.

The output of the inverse Laplace transform consists of:

a. Real part of root (σ_t or p_i).

b. Imaginary part of root (ω_t).

c. Multiplicity of root (K).

d. Inverse Laplace transform equation of each individual partial fraction.

4. Restriction.

a. Maximum order of numerator is 51.

b. Maximum order of denominator is 52.

c. Maximum orders of K, multiplicity of roots are:

Roots at origin, $0 \leq K \leq 3$.

Roots not at origin on real axis, $0 \leq K \leq 2$.

Roots in complex plane, $0 \leq K \leq 1$.

4.2 Plotting Routine for Inverse Laplace Transform.

1. General Description.

a. Computes the numerical values of the inverse Laplace transform as a function of time.

b. Produces a graphical display using line printer.

2. Input.

Description		Columns	Data
(a)	Identification	1-80 (Alphanumeric)	Identification of problem
(b)	Running time of problem	1-15 (Floating Pt.)	Start time
		16-30 (Floating Pt.)	Stop time
	Scaling of Output Response*	(31-40) (Floating Pt.)	Minimum value
		(41-50) (Floating Pt.)	Maximum

3. Output.

Produces a time response, tabulated output and graphical display using on-line printer consisting of:

a. Start and stop time of computation.

b. Automatic or preselected scaled ordinate axis with values.

*The output response is automatically scaled if minimum and maximum values are not specified.

c. Automatically scaled time axis with values.

d. Response plots of tabulated output.

4. Restrictions.

a. Cannot exceed 15 time intervals of a particular problem without reading in coefficients again.

b. Independent variable time must be positive (trivial requirement of physical systems).

4.3 Placement of Data Cards.

Group No.	Content Description	Number of Cards
1	Identification of Data	1
2	Time (start/stop , Min/Max Val)	Max 15
3	Blank Card	
4	Power; Coefficients of Numerator	Max 51
5	Power; Coefficients of Denominator	Max 52
6	Roots of Denominator, Multiplicity	As many as needed
7	Blank Card	

Data placement is repeated for next problem making certain that the blank cards are properly placed.

5. CAPABILITY OF THE PROGRAM

The type of input forcing function that the system is subjected to must be included in the ratio of polynomials of Equation (4). A new polynomial ratio is

formed by this multiplication.

$$g(s) = g_1(s)g_2(s),$$

where $g_1(s)$ is the system input forcing function, and $g_2(s)$ is the system transfer function.

The program is capable of handling the following system input forcing functions, $g_1(s)$.

F(t)	Type of Input	$g_1(s)$
δ	impulse	1
1	step	$1/s$
t	rate	$1/s^2$
$t^2/2$	parabolic	$1/s^3$
e^{-at}	exponential lag	$\dfrac{1}{s + a}$
$\dfrac{t}{T^2} e^{-t/T}$	2nd order exponential lag	$\dfrac{1}{(s + a)^2}$
$\omega(1+a^2\omega^2)^{1/2} \sin(\omega t+\psi)$, $\psi = \tan^{-1}(a\omega)$	Undamped phase shifted sinusoid	$\dfrac{1 + as}{1 + \dfrac{s^2}{\omega^2}}$
$\omega\left[\dfrac{1 - 2a\zeta\omega + a^2\omega^2}{1 - \zeta^2}\right]^{1/2} e^{-\zeta\omega t} \sin(\omega\sqrt{1-\zeta^2}\,t+\psi)$, $\psi = \tan^{-1}\dfrac{(a\omega\sqrt{1-\zeta^2})}{1 - a\zeta\omega}$	Damped phase shifted sinusoid	$\dfrac{1 + as}{1 + \dfrac{2\zeta s}{\omega} + \dfrac{s^2}{\omega^2}}$

The inverse Laplace transform digital program is capable of solving any system transfer function being acted on by a forcing function which can be broken down into the following partial fraction forms:

Location of Denominator Roots (Poles)	Partial Fraction Form
Origin	$\frac{c}{s}, \frac{c}{s^2}, \frac{c}{s^3}$
Real Axis	$\frac{c}{s+a}, \frac{c}{(s+a)^2}$
Complex Plane	$\frac{\vec{c}}{(s^2 + 2\sigma\omega s + \omega^2)}$

The maximum order of the numerator of g(s) can not exceed 51 and the maximum order of the denominator of g(s) can not exceed 52.

6. EXAMPLE PROBLEMS

Three transfer functions, $g_2(s)$, are subjected to various input forcing functions, $g_1(s)$, to demonstrate the utility of the program. Tables 1, 2, and 3 demonstrate the mirror of the input for each system subjected to one specific input forcing function.

EXAMPLE 1: $$g_2(s) = \frac{g_{2N}}{g_{2D}}$$

where: $$\frac{g_{2N}}{g_{2D}} = \frac{3s + 11}{s^2 + 8s + 15}$$

$$g_1(s) = \frac{1}{s}\ ,\ \frac{1}{s + 1}\ ,\ \frac{1 + 0.5s}{1 + 0.3s + 0.25s^2}$$

EXAMPLE 2:

$$\frac{g_{2N}}{g_{2D1} + g_{2D2} + g_{2D3} + g_{2D4} + g_{2D5} + g_{2D6}}$$

where: $g_{2N} = 1.34 \times 10^{-3}s^2 + 2.067 \times 10^{-1}s + 1.0$

$g_{2D1} = 6.9981 \times 10^{-7}s^5$

$g_{2D2} = 1.15529 \times 10^{-4}s^4$

$g_{2D3} = 1.68 \times 10^{-3}s^3$

$g_{2D4} = 3.828 \times 10^{-3}s^2$

$g_{2D5} = 2.593 \times 10^{-1}s$

$g_{2D6} = 1.0$

$$g_1(s) = \frac{1}{s}, \frac{1}{s+1}, \frac{1}{(s+1)^2}, \frac{1}{1 + 0.2666s + 0.111s^2}$$

EXAMPLE 3:

$$g_2(s) = \frac{g_{2N1} + g_{2N2} + g_{2N3} + g_{2N4} + g_{2N5}}{g_{2D1} + g_{2D2} + g_{2D3} + g_{2D4} + g_{2D5}}$$

where: $g_{2N1} = 2.46665 \times 10^{1}s^4$

$g_{2N2} = 6.017766 \times 10^{2}s^3$

$g_{2N3} = 7.137658 \times 10^{5}s^2$

$g_{2N4} = 1.3620729 \times 10^{7}s$

$g_{2N5} = 4.7818957 \times 10^{8}$

$g_{2D1} = 1.0s^4$

$g_{2D2} = 9.2 \times 10^{0}s^3$

$g_{2D3} = 2.60192 \times 10^{4}s^2$

$$g_{2D4} = 1.5488 \times 10^5 s$$

$$g_{2D5} = 1.024 \times 10^7$$

$$g_1(s) = \frac{1}{s^2}$$

7. CONCLUSIONS

The program described in this technical memorandum for computing the inverse Laplace transforms and plotting their time history response as a function of real time has the following merits:

1. Uncomplicated data loading.

2. Produces equations in exact inverse Laplace form.

3. Produces an accurate numerical printout of selected points.

4. Produces a graphical display for visual description of system response.

The main objective of this program, which is to provide expanded systems analysis capability, has been realized. Transient solutions of high-order differential equations being driven by a wide variety of forcing functions can be generated rapidly.

TABLE 1. INPUT FOR EXAMPLE 1

$$g_1(s) = \frac{1}{s}$$

Mirror Of Input			Columns	Description
Input for Example 1.			(1-80)	Identification
0.000	2.500		(1-10)(11-20)	Running Time Start-Stop
0000			(1-80)	Separator (Blank)
1	3.000		(1-5)(6-20)	Degree-Coefficient of Numerator
0	11.000			
3	1.000		(1-5)(6-20)	Degree-Coefficient of Denominator
2	8.000			
1	15.000			
0	0.000			
5.000	0.000	1	(1-15)(16-30) (31-45)	Pole Root Real Part-Imaginary Part-Multiplicity
3.000	0.000	1		
0000			(1-80)	Separator (Blank)

TABLE 2. INPUT FOR EXAMPLE 2*

$$g_1(s) = \frac{1}{s}$$

Mirror of Input			Columns	Description
Input for Example 2.			(1-80)	Identification
0.000	5.000		(1-10)(11-20)	Running Time Start-Stop
0.000	1.200			
0.000	.500			
.500	2.000			
0000			(1-80)	Separator (Blank)
2	1.340E-03		(1-5)(6-20)	Degree-Coefficient of Numerator
1	2.067E-01			
0	1.000			
6	6.9981E-07		(1-5)(6-20)	Degree-Coefficient of Denominator
5	1.1552908E-04			
4	1.68E-03			
3	3.828E-02			
2	2.593E-01			
1	1.000			
0	0.000			
4.1258865	4.2939554	1	(1-15)(16-30) (31-45)	Pole Root Real-Part-Imaginary-Multiplicity
2.6595745	16.89876	1		
151.5151	0.000	1		
0000			(1-80)	Separator (Blank)

*See example on page 165 of Reference 6.

6 Nixon, Floyd E., Principles of Automatic Controls, Prentice Hall, Inc., New York, 1954.

TABLE 3. INPUT FOR EXAMPLE 3

$$g_1(s) = \frac{1}{s^2}$$

Mirror Of Input			Columns	Description
Input for Example 3.			(1-80)	Identification
0.000 0.000 0.000	.1000 .0100 10.000		(1-10)(11-20)	Running Time Start-Stop
0000			(1-80)	Separator (Blank)
4 3 2 1 0	24.6665 601.7766 7.137658E05 1.3620729E07 4.7818957E08		(1-5)(6-20)	Degree-Coefficient of Numerator
6 5 4 3 2 1 0	1.000 9.200 2.60192E04 1.5488E05 1.024E07 0.000 0.000		(1-5)(6-20)	Degree-Coefficient of Denominator
3.000 1.600	19.77372 159.9992	1 1	(1-15)(16-30) (31-45)	Pole Root Real Part-Imaginary Part-Multiplicity
0000			(1-80)	Separator (Blank)

APPENDIX A

LISTING OF SOURCE DECK

```
C   CANNOT EXCEEOTHE FOLLOWING...3RO OROER POLE AT ORIGIN,2NO ORDER POLE
C   NOT AT ORIGIN, 1ST OROER COMPLEX POLE. ACCEPTS STABLE ANO UNSTABL
C   SYSTEMS. ACCEPTS UP TO 15 COMPUTING INTERVALS.
C     PUT 8LANK CARO AFTER PRESELECTEO TIME AND AXIS VALUES IN DATA
C   PUT 8LANK CARD AFTER EACH PROBLEM IN OATA
C   FINOING THE INVERSE LAPLACE TRANSFORM OF A QUOTIENT OF TWO
C   POLYNOMIALS  (INVERSE TRANSFORM PARAMETERS)
        REAL  PDTOP(60)
        REAL COEF,NO,N1,01,02,03,COD(60),CON(60),DIF
        REAL RE(60),IM(60)
        REAL N(51),NP(50),0(52),OP(51),OPP(50),DPPP(49)
        INTEGER DN,DNP1,DD,ODP1,I,K
        REAL PALPH(60),P8ETA(60),PAOR8(60),PTHRC(60)
        INTEGER KK(60)
        INTEGER  COMENT(20)
        INTEGER  ITT
        REAL TSTA(15),TSTOP(15),YSTA(15),YSTOP(15)
 90     FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,3H)+(,E18.7,3H)*T)
 91     FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,15H)*EXP(-ALPHA*T))
 92     FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,3H)+(,E18.7,5H)*T+(,
       1E18.7,5H)*T*T)
 93     FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,17H)*EXP(-ALPHA*T)+(,E18.7,
       117H)*T*EXP(-ALPHA*T))
 95     FORMAT(1H ,1P2E18.7,I4,5X,
       11H(,E18.7,30H)*EXP(-ALPHA*T)*SIN((8ETA*T)+(,E18.7,2H)))
        ICSE=1
 999    CONTINUE
        WRITE(6,150)ICSE
 150    FORMAT(1H1,//////////////////////////////,60X,12HPRO8LEM NUM.,I4)
        READ(5,888) COMENT
 888    FORMAT(20A4)
        WRITE(6,777) COMENT
 777    FORMAT(1H ,//,20X,20A4)
 99     JTA=1
9999    CONTINUE
C   READ REAL TIME COMPUTING INTERVALS
        READ(5,101) TSTA(JTA),TSTOP(JTA),YSTA(JTA),YSTOP(JTA)
 101    FORMAT(4F10.0)
        IF(TSTA(JTA).EQ.0.0.AND.TSTOP(JTA).EQ.0.0 )GOTO9998
        JTA=JTA+1
        IF(JTA.GT.15) GO TO 99
        GO TO 9999
9998    CONTINUE
C     CLEAR PARAMETERS
        DO 500 LCR=1,60
        RE(LCR)=0.0
        CON(LCR)=0.0
        IM(LCR)=0.0
        COD(LCR)=0.0
        PDTOP(LCR)=0.0
        PALPH(LCR)=0.0
        KK(LCR)=0.0
        PAOR8(LCR)=0.0
        P8ETA(LCR)=0.0
 500    PTHRC(LCR)=0.0
        DO 510 M=1,52
        0(M)=0.0
        COO(M)=0.0
 510    CONTINUE
```

```
      DO 511 M=1,51
      CON(M)=0.0
      RE(M)=0.0
      IM(M)=0.0
      N(M)=0.0
      DP(M)=0.0
 511  CONTINUE
      DO 512 M=1,50
      NP(M)=0.0    0
      DPP(M)=0.0
 512  CONTINUE
      DO 513 M=1,49
      DPPP(M)=0.0
 513  CONTINUE
      WRITE(6,15)
 15   FORMAT(1H1,7X,5HORDER,11X,4HCOEF,/)
C     READ COEF. OF HIGHEST POWER OF S IN NUMERATOR
      READ(5,1)K,COEF
 1    FORMAT(I5,F15.0)
      WRITE(6,111)
 111  FORMAT(1H ,7X,18HTERMS IN NUMERATOR,/)
      WRITE(6,19) K,COEF
19    FORMAT(1H ,5X,I4,5X,1P2E18.7)
      III=0
      KN=K+1
      I=51-K
      KP1=K+1
      CON(KP1)=COEF
      DN=I
      DNP1=DN+1
      N(I)=COEF
C     READ REST OF COEF. OF NUMERATOR
      GO TO 60
    2 READ(5,1)K,COEF
      WRITE(6,19) K,COEF
      I=51-K
      KP1=K+1
      CON(KP1)=COEF
      N(I)=COEF
C   ZERO COEF CHECK,START COEF OF DENOMINATOR
60    IF(K.NE.0)GO TO 2
C     READ COEF OF HIGH. POWER OF S IN DEN.
      WRITE(6,112)
 112  FORMAT(1H ,//,7X,20HTERMS IN DENOMINATOR,/)
      READ(5,1)K,COEF
      WRITE(6,19) K,COEF
      KD=K+1
      I=52-K
      KP1=K+1
      COD(KP1)=COEF
C     DEGREE OF DEN.
      DD=I
      DDP1=DD+1
      D(I)=COEF
C     REST OF COEF. OF DEN.
    3 READ(5,1)K,COEF
      WRITE(6,19) K,COEF
      I=52-K
      KP1=K+1
      COD(KP1)=COEF
```

```
      D(I)=COEF
      IF(K.NE.0)GO TO 3
C  END READING OF COEF,START DERIVATIVE CALCULATIONS
C     DERIVATIVE OF DEN.
      DO 4 I=DD,51
      AI=52-I
    4 DP(I)=AI*D(I)
C     2ND DERIVATIVE OF DEN.
      DO 5 I=DD,50
      AI=51-I
    5 DPP(I)=AI*DP(I)
C     3RD DER. OF DEN.
      DO 6 I=DD,49
      AI=50-I
    6 DPPP(I)=AI*DPP(I)
C     GET DER. OF NUM.
      DO 7 I=DN,50
      AI=51-I
    7 NP(I)=AI*N(I)
      WRITE(6,115)
115   FORMAT(1H ,////,8X,5HALPHA,12X,4HBETA,10X,1HK,19X,
     120HINVERSE LAPLACE FORM,//)
C     FIRST ORDER POLE AT ORIGIN
      IF(D(52).EQ.0.0.AND.D(51).NE.0.0)GO TO 11
      GO TO 10
   11 ALPHA=0.0
      BETA=0.0
      K=1
      AORB=N(51)/D(51)
      THORC=0.0
      WRITE(6,90)ALPHA,BETA,K,AORB,THORC
      III=III+1
      PALPH(III)=ALPHA$  KK(III)=K$ PAORB(III)=AORB$
      PBETA(III)=BETA$  PTHRC(III)=THORC
      GO TO 40
 10   IF(D(52).EQ.0.0.AND.D(51).EQ.0.0.AND.D(50).NE.0.0)GOTO13
      GOTO12
C     SECOND ORDER POLE AT ORIGIN
   13 ALPHA=0.0
      BETA=0.0
      K=2
      AORB=(D(50)*N(50)-D(49)*N(51))/D(50)**2
      THORC=N(51)/D(50)
      WRITE(6,90)ALPHA,BETA,K,AORB,THORC
      III=III+1
      PALPH(III)=ALPHA$  KK(III)=K$ PAORB(III)=AORB$
      PBETA(III)=BETA$  PTHRC(III)=THORC
      GO TO 40
 12   IF(D(52).EQ.0.0.AND.D(51).EQ.0.0.AND.D(50).EQ.0.0)GOTO14
C     NO POLES AT ORIGIN - GO TO READ POLE CD.
      GO TO 40
C     THIRD ORDER POLE AT ORIGIN
 14   ALPHA=0.0
      BETA=0.0
      K=3
      AORB=(D(49)*D(49)*N(49)-D(49)*D(47)*N(51)-D(49)*D(48)*N(50)
     1+N(51)*D(48)*D(48))/(D(49)*D(49)*D(49))
      THORC=(D(49)*N(50)-D(48)*N(51))/(D(49)*D(49))
      DTOP=N(51)/(2.0*D(49))
      WRITE(6,92)ALPHA,BETA,K,AORB,THORC,DTOP
```

```
      III=III+1
      PALPH(III)=ALPHA$ KK(III)=K$PADRB(III)=ADRB
      PBETA(III)=BETA$ PTHRC(III)=THDRC$ PDTOP(III)=DTOP
C     POLES AT ORIGIN TESTED FOR,READ OTHER POLES
   40 READ(5,2D)ALPHA,BETA,K
 20   FORMAT(2F15.D,I5)
      WRITE(6,9) ALPHA,BETA,K
    9 FORMAT(1H0,1P2E18.7,I4,2E18.7)
      IF(ALPHA.EQ.D.O.AND.BETA.EQ.D.) GD TD 1D9
      IF(BETA.NE.D.D)GDTD5D
      IF(K.NE.2)GO TO 3D
      ND=N(DN)
      DD 21 I=DNP1,51
   21 ND=-ALPHA*NO+N(I)
      IF(DNP1.GT.51)NO=N(DN)
      N1=NP(DN)
      DO 22 I=DNP1,50
   22 N1=-ALPHA*N1+NP(I)
      IF(DNP1.GT.5D)N1=NP(DN)
      D2=DPP(DD)
      DD 23 I=DDP1,5D
   23 D2=-ALPHA*D2+DPP(I)
      IF(DDP1.GT.5D)D2=DPP(DD)
      D3=DPPP(DD)
      DD 24 I=DDP1,49
   24 D3=-ALPHA*D3+DPPP(I)
      IF(DDP1.GT.49)D3=DPPP(DD)
      ADRB=(D2/2.D*N1-ND*D3/6.D)/(D2**2/4.0)
      THDRC=2.D*ND/D2
      WRITE(6,93)ALPHA,BETA,K,AORB,THDRC
      III=III+1
      PALPH(III)=ALPHA$  KK(III)=K$ PAORB(III)=AORB$
      PBETA(III)=BETA$  PTHRC(III)=THORC
      GO TO 40
   3D IF(BETA.NE.D.0)GO TD 5D
      NO=N(DN)
      DD 31 I=DNP1,51
   31 ND=-ALPHA*NO+N(I)
      IF(DNP1.GT.51)ND=N(DN)
      D1=DP(DD)
      DO 32 I=DDP1,51
   32 D1=-ALPHA*D1+DP(I)
      IF(DDP1.GT.51)D1=DP(DD)
      ADRB=ND/D1
      THDRC=D.D
      WRITE(6,91)ALPHA,BETA,K,ADRB,THORC
      III=III+1
      PALPH(III)=ALPHA $ KK(III)=K $  PADRB(III)=ADRB
      PBETA(III)=BETA $ PTHRC(III)=THORC
      GD TD 4D
C   COMPLEX PDLES
 50   TDTNR=CDN(1)$ TDTNI=D.$ TDTDR=CDD(2)$ TDTDI=0.D
      A=-ALPHA$ B=BETA$  AX=1.D$ AY=0.D
      DD 3DD IR=1,KD
      RE(IR)=A*AX-B*AY
      IM(IR)=B*AX+A*AY
      AX=RE(IR)
      AY=IM(IR)
 3D0  CONTINUE
      DD 301 IN=1,KN
```

```
      INP1=IN+1
      IF(INP1. GT.KN)GO TO 305
      TOTNR=CON(IN+1)*RE(IN)+TOTNR
  301 TOTNI=CON(IN+1)*IM(IN)+TOTNI
  305 CONTINUE
      DO 302 ID=1,KD
      DIF=ID+1
      IDP2=ID+2
      IF(IDP2.GT.KD)GO TO 306
      TOTDR=COD(ID+2)*RE(ID)*DIF+TOTDR
  302 TOTDI=COD(ID+2)*IM(ID)*DIF+TOTDI
  306 CONTINUE
      AMD=SQRT(TOTDR**2+(TOTDI**2))
      AMN=SQRT(TOTNR**2+(TOTNI**2))
      ANGD=ATAN2(TOTDR,TOTDI)
      ANGN=ATAN2(TOTNR,TOTNI)
      AORB=2.0*AMN/AMD
      THORC=ANGN-ANGD
      YIM=AORB*SIN(THORC)
      YRE= AORB*COS(THORC)
      THORC=ATAN2(YRE,YIM)
C     IF IN FIRST OR FOURTH QUAD,LEAVE IT
C     OTHER WISE ADD OR SUB PI TO GET SUCH, AND REVERSE
C     SIGN TO COMPENSATE
      IF(ABS(THORC).LE.1.57079632) GO TO 453
      IF(THORC)450,451,452
450   THORC=THORC+3.14159265
      GOTO 451
452   THORC=THORC-3.1415926
451   AORB=-AORB
  453 CONTINUE
      WRITE(6,95)ALPHA,BETA,K,AORB,THORC
      III=III+1
      PALPH(III)=ALPHA$  KK(III)=K$ PAORB(III)=AORB$
      PBETA(III)=BETA$   PTHRC(III)=THORC
      GO TO 40
  109 CONTINUE
      DO 1100 ITT=1,15
      CALL PLOT(PALPH,PBETA,KK,PAORB,PTHRC,TSTA,TSTOP,ITT,PDTOP,
     1YSTA,YSTOP)
      IF(TSTA(ITT+1).EQ.0.0.AND.TSTOP(ITT+1).EQ.0.0)GO TO 1200
1100  CONTINUE
1200  CONTINUE
      ICSE=ICSE+1
      GOTO999
      END
C    END COMPUTATIONS,START GRAPHICAL DISPLAY
C     PROGRAM TO CHOOSE TIME INTERVAL AND GRAPH RESULTS
      SUBROUTINE PLOT(PALPH,PBETA,KK,PAORB,PTHRC,TSTA,TSTOP,
     1ITT,PDTOP,YSTA,YSTOP)
      REAL TSTA(15),TSTOP(15),YSTA(15),YSTOP(15)
      INTEGER  ITT
      REAL VALT(15)
      REAL  FT(15),TIME(15)
      REAL PALPH(60),PBETA(60),PAORB(60),PTHRC(60)
      REAL  PDTOP(60)
      INTEGER KK(60),K(60),I,L,J
      REAL ALPHA(60),BETA(60),AORB(60),THORC(60),E,F,T,TF,FP
      REAL ALPHAS,ALPHAL,BETAS,BETAL,DELTAT,TEND,DELTA
      INTEGER IS
```

```
      REAL   GRAPH(60,130)
      DATA BLANK/1H /, DOT/1H./, X/1HX/
      TEND=TSTOP(ITT)
      BPO=-9999999999.
      SNE=9999999999.  $ZAP=SNE
      AI=0.$ BJ=0.$  FI=0.0$  FJ=0.0
      DO 430 NCL=1,15
      FT(NCL)=0.0
      TIME(NCL)=0.0
  430 VALT(NCL)=0.0
      FT(1)=1.0$ FT(2)=2.0$ FT(3)=3.0$ FT(4)=4.0$ FT(5)=5.0
      VALT(1)=TSTA(ITT)
      GEST=0.0
      BEST=0.0
      TEST=0.0
C     ARRAY FOR GRAPH
      DO 612 I=1,60
      DO 612 J=1,130
612   GRAPH(I,J)=BLANK
C     DOTS FOR AXES
      DO 667 I=10,60
 667  GRAPH(I,10)=DOT
      DO 670 I=10,12
      DO 670 J=10,130,10
 670  GRAPH(I,J)=DOT
      DO 668 J=10,130
 668  GRAPH(10,J)=DOT
C     WRITE HEADING
      WRITE(6,9998)
9998  FORMAT(1H1,15X,1HT,13X,3HFOT,10X,5HDELTA,7X,6HDELTAT,//)
      DO 501 L=1,60
      M=L
      ALPHA(M)=PALPH(M)
      BETA(M)=PBETA(M)
      K(M)=KK(M)
      AORB(M)=PAORB(M)
      THORC(M)=PTHRC(M)
      IF(K(M).EQ.0) GO TO 64
 501  CONTINUE
64    L=L-1
C     GET ALPHAS,ALPHAL,IS,BETAS,BETAL
      IS=1
      ALPHAS=ABS(ALPHA(1))
      ALPHAL=ABS(ALPHA(1))
      BETAS=ABS(BETA(1))
      BETAL=ABS(BETA(1))
      DO 23 I=2,L
      IF(ALPHAS.LT.ABS(ALPHA(I)))GO TO 24
      ALPHAS=ABS(ALPHA(I))
      IS=I
24    ALPHAL=AMAX1(ALPHAL,ABS(ALPHA(I)))
      IF(BETA(I).EQ.0.0)GO TO 23
      BETAS=AMIN1(BETAS,ABS(BETA(I)))
      BETAL=AMAX1(BETAL,ABS(BETA(I)))
23    CONTINUE
 327  CONTINUE
C   FIND INITIAL STEP SIZE,DELTAT
      IF(ALPHAL.GT.BETAL)GO TO 25
      DELTAT=.1/BETAL
      GO TO 26
```

```
 25     DELTAT=.1/ALPHAL
 26     CONTINUE
        T=TSTA(ITT)
C       EVALUATE FUNCTION
 67     F=0.0
        DO 78 I=1,L
        IF(BETA(I))80,81,80
C       COMPLEX-NOT ZERO BETA
80      F=F+AORB(I)*EXP(-ALPHA(I)*T)*SIN(BETA(I)*T+THORC(I))
        GO TO 78
C       REAL - BETA ZERO
81      E=EXP(-ALPHA(I)*T)
        F=F+AORB(I)*E
        IF(K(I).EQ.2)F=F+THORC(I)*T*E
        IF(K(I).EQ.3)F=F+PDTOP(I)*T*T*E
78      CONTINUE
        IF(YSTA(ITT).NE.0.0.AND.YSTOP(ITT).NE.0.0)GOTO326
        IF(TEST.EQ.1.0) GO TO 326
        DO 411 NTE=1,4
        FT(NTE)=FT(NTE +1)
        TIME(NTE)=TIME(NTE + 1)
 411    CONTINUE
        FT(5)=F
        TIME(5)=T
        IF(T.GT.TSTOP(ITT))GO TO 333
C    DETERMINE SCALING OF AXIS
        SNE=AMIN1(SNE,F)
        BPO=AMAX1(BPO,F)
        DO 412 NTE=2,5
        DJAA=ABS(FT(NTE-1)-FT(NTE))
        IF(DJAA.GT.(ABS(FT(1))*.00000001))GOTO 49
 412    CONTINUE
        TEND=TIME(1)$ TEST=1.0
        GO TO 327
 333    TEND=TSTOP(ITT)$ TEST=1.0
        GO TO 327
 326    CONTINUE
        IF(BEST.EQ.1.0) GO TO 399
        IF(YSTA(ITT).EQ.0.0.AND.YSTOP(ITT).EQ.0.0)GOTO0305
        BPO=YSTOP(ITT)
        SNE=YSTA(ITT)
 305    CONTINUE
        FI=50.0/(BPO-SNE)
        FJ=120.0/(TEND-TSTA(ITT))
        BEST=1.0
 399    CONTINUE
        TF=F
        WRITE(6,72)T,TF,DELTA,DELTAT
 72     FORMAT(1H ,4X,2F15.4,6X,2F9.4)
        AI=(FI*(TF-SNE))+10.0
        IF(AI.GT.100000.)GOTO450
        I=IFIX(AI)
        IF(GEST.EQ.1.0)GOTO39
        ZPT=BPO
        SZRO=(BPO-SNE)/50.0
        DO 350 KT=1,50
        ZAP=AMIN1(ZAP,ABS(ZPT))
        ZPT=ZPT-SZRO
 350    CONTINUE
        ZI=(FI*(ZAP-SNE))+10.0
```

```
      JA=IFIX(ZI)
      DO 92 J=10,130,5
 92   GRAPH(JA,J)=OOT
      GEST=1
 39   CONTINUE
      8J=(FJ*(T-TSTA(ITT)))+10.0
      IF(8J.GT.100000.00)GOTO 450
      J=IFIX(BJ)
      IF(TF.EQ.SNE) I=10
      IF(TF.EQ.BPO)  I=60
      IF(T.EQ.TSTA(ITT))  J=10
      IF(T.EQ.TENO)  J=130
      IF(I.GE.10.ANO.I.LE.60.ANO.J.GE.10.AND.J.LE.130)GO TO 400
      GO TO 450
 400  GRAPH(I,J)=X
 450  CONTINUE
      IF(T.GE.TEND)GO TO 666
C     FINO OERIVATIVE AT T=T AND T=T+DELTAT
49    FP=0.0
      00 178 I=1,L
      E=EXP(-ALPHA(I)*T)
      IF(8ETA(I))180,181,180
C     COMPLEX - 8ETA NOT ZERO
180   FP=FP+AOR8(I)*E*COS(BETA(I)*T+THORC(I))*8ETA(I)
     1 -AOR8(I)*ALPHA(I)*E*SIN(8ETA(I)*T+THORC(I))
      GO TO 178
C     REAL - 8ETA ZERO
181   FP=FP-ALPHA(I)*AOR8(I)*E
      IF(K(I).EQ.2)FP=FP+THORC(I)*E*(1.0-ALPHA(I)*T)
      IF(K(I).EQ.3)FP=FP+POTOP(I)*E*(2.0*T-ALPHA(I)*T*T)
178   CONTINUE
      AX=2.0*FP
      Y=T+DELTAT
      FP=0.0
      DO 278 I=1,L
      E=EXP(-ALPHA(I)*Y)
      IF(8ETA(I))280,281,280
280   FP=FP+AORB(I)*E*COS(8ETA(I)*Y+THORC(I))*BETA(I)
     1 -AOR8(I)*ALPHA(I)*E*SIN(8ETA(I)*Y+THORC(I))
      GO TO 278
281   FP=FP-ALPHA(I)*AOR8(I)*E
      IF(K(I).EQ.2)FP=FP+THORC(I)*E*(1.0-ALPHA(I)*Y)
      IF(K(I).EQ.3)FP=FP+PDTOP(I)*E*(2.0*Y-ALPHA(I)*Y*Y)
278   CONTINUE
      FP=2.0*FP
      OELTA=A8S(ATAN(AX)-ATAN(FP))
C     END DERV. EVAL.
      IF(DELTA.GE.0.05)GO TO 50
      T=T+DELTAT
      IF(T.GE.TEND)T=TENO
      DELTAT=1.500*OELTAT
      GO TO 67
 50   IF(OELTA.GE.0.15.AND.DELTAT.GE.(TSTOP(ITT)-TSTA(ITT))/
     1120.0)GO TO 60
      T=T+DELTAT
      IF(T.GE.TENO)T=TENO
      GO TO 67
60    DELTAT=DELTAT/2.0
      GO TO 49
 666  WRITE(6,120)TSTA(ITT),TENO,TSTOP(ITT)
```

```
 120  FORMAT(1H1,11HSTART TIME=,F10.4,10X,17HACTUAL STOP TIME=,F10.4,
     110X,19HPROPOSED STOP TIME=,F10.4)
      WRITE(6,8)
 8    FORMAT(1H ,4X,4HF(T),/)
      VALY=BPO
      SCAY=(BPO-SNE)/50.0
      DO 29 I1=1,51
      I=61-I1
      WRITE(6,30)VALY,(GRAPH(I,J),J=10,130)
      VALY=VALY-SCAY
 29   CONTINUE
 30   FORMAT(1H ,F8.3,1X,121A1)
      SCAX=(TEND-VALT(1))/12.00
      DO 680 LAX=2,13
 680  VALT(LAX)=VALT(LAX-1)+SCAX
      WRITE(6,5)(VALT(LAX),LAX=1,13)
 5    FORMAT(1H ,2X,12F10.3,F9.3)
      WRITE(6,7)
 7    FORMAT(1H ,/,50X,14HREAL TIME(SEC))
C     TAKE ANOTHER SET OF DATA
      RETURN
      END
```

APPENDIX B

FLOW CHART OF PROGRAM

```
                                  (ENTRANCE)
                                      (
                                      I
*************************************************************************
*C                                                                      *
*C  CANNOT EXCEEDTHE FOLLOW(NG...3RD ORDER POLE AT ORIGIN,2ND OROER POLE *
*C  NOT AT ORIGIN, 1ST OROER COMPLEX POLE. ACCEPTS STABLE AND UNSTABL   *
*C  SYSTEMS. ACCEPTS UP TO 15 COMPUTING INTERVALS.                      *
*C   PUT BLANK CARD AFTER PRESELECTEO TIME AND AXIS VALUES IN DATA      *
*C  PUT BLANK CARO AFTER EACH PROBLEM IN DATA                           *
*C  FINOING (HE INVERSE LAPLACE TRANSFORM OF A QUOTIENT OF TWO          *
*C  POLYNOMIALS  (INVERSE TRANSFORM PARAMETERS)                         *
*       REAL  PDTOP(60)                                                 *
*       REAL COEF,NO,N1,D1,D2,D3,COD(60),CON(60),OIF                    *
*       REAL RE(60),IM(60)                                              *
*       REAL N(51),NP(50),O(52),DP(51),OPP(50),DPPP(49)                 *
*       INTEGER ON,DNP1,DD,OOP1,I,K                                     *
*       REAL PALPH(60),P8ETA(60),PAOR8(60),PTHRC(60)                    *
*       INTEGER KK(60)                                                  *
*       INTEGER  COMENT(20)                                             *
*       INTEGER  ITT                                                    *
*       REAL TSTA(15),TSTOP(15),YSTA(15),YSTOP(15)                      *
* 90    FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,3H)+(,E18.7,3H)*T)          *
* 91    FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,15H)*EXP(-ALPHA*T))         *
* 92    FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,3H)+(,E18.7,5H)*T+(,        *
*      1E18.7,5H)*T*T)                                                  *
* 93    FORMAT(1H ,1P2E18.7,I4,5X,1H(,E18.7,17H)*EXP(-ALPHA*T)+(,E18.7, *
*      117H)*T*EXP(-ALPHA*T))                                           *
* 95    FORMAT(1H ,1P2E18.7,I4,5X,                                      *
*      11H(,E18.7,30H)*EXP(-ALPHA*T)*SIN((8ETA*T)+(,E18.7,2H)))         *
*       ICSE=1                                                          *
*************************************************************************
                                      I
                                      0<------------------------------------0
                                      I                                     I
*************************************************************************   I
* 999   CONTINUE                                                        *   I
*       WRITE(6,150)ICSE                                                *   I
* 150   FORMAT(1H1,///////////////////////////////,60X,12HPROBLEM NUM.,I4) * I
*       REAO(5,888) COMENT                                              *   I
* 888   FORMAT(20A4)                                                    *   I
*       WRITE(6,777) COMENT                                             *   I
* 777   FORMAT(1H ,//,20X,20A4)                                         *   I
*************************************************************************   I
                                      I                                     I
                                      0<------------------------------------I-------0
                                      I                                     I       I
*************************************************************************   I       I
* 99    JTA=1                                                           *   I       I
*************************************************************************   I       I
                                      I                                     I       I
                                      0<------------------------------------I-------I-------0
                                      (                                     I       I       I
*************************************************************************   I       I       I
*9999   CONTINUE                                                        *   I       I       I
*C  READ REAL TIME COMPUTING (INTERVALS                                 *   I       I       I
*       REAO(5,101) TSTA(JTA),TSTOP(JTA),YSTA(JTA),YSTOP(JTA)           *   I       I       I
* 101   FORMAT(4F10.0)                                                  *   I       I       I
*************************************************************************   I       I       I
                                      I                                     I       I       I
```

```
      IF(TSTA(JTA).EQ.0.0.AND.TSTOP(JTA).EQ.0.0 )GOTO9998

      JTA=JTA+1

      IF(JTA.GT.15) GO TO 99

      GO TO 9999

9998  CONTINUE
C   CLEAR PARAMETERS

      DO 500 LCR=1,60

      RE(LCR)=0.0
      CCN(LCR)=0.0
      IM(LCR)=0.0
      COD(LCR)=0.0
      PDTOP(LCR)=0.0
      PALPH(LCR)=0.0
      KK(LCR)=0.0
      PAORB(LCR)=0.0
      PBETA(LCR)=0.0
```

```
I                                      I                              I
I                                      I                              I
I                                      I                              I
I              ***************************************************   I
---------------* 500  PTHRC(LCR)=0.0                              *   I
               ***************************************************   I
                                       I                              I
                                       I                              I
                                       I                              I
               ***************************************************   I
I--------------*      DO 510 M=1,52                               *   I
I              ***************************************************   I
I                                      I                              I
I                                      I                              I
I                                      I                              I
I              ***************************************************   I
I              *      D(M)=0.0                                    *   I
I              *      COO(M)=0.0                                  *   I
I              ***************************************************   I
I                                      I                              I
I                                      I                              I
I                                      I                              I
I              ***************************************************   I
---------------* 510  CONTINUE                                    *   I
               ***************************************************   I
                                       I                              I
                                       I                              I
                                       I                              I
               ***************************************************   I
I--------------*      DO 511 M=1,51                               *   I
I              ***************************************************   I
I                                      I                              I
I                                      I                              I
I                                      I                              I
I              ***************************************************   I
I              *      CON(M)=0.0                                  *   I
I              *      RE(M)=0.0                                   *   I
I              *      IM(M)=0.0                                   *   I
I              *      N(M)=0.0                                    *   I
I              *      OP(M)=0.0                                   *   I
I              ***************************************************   I
I                                      I                              I
I                                      I                              I
I                                      I                              I
I              ***************************************************   I
---------------* 511  CONTINUE                                    *   I
               ***************************************************   I
                                       I                              I
                                       I                              I
                                       I                              I
               ***************************************************   I
I--------------*      DO 512 M=1,50                               *   I
I              ***************************************************   I
I                                      I                              I
```

```
      NP(M)=0.0    0
      DPP(M)=0.0

 512  CONTINUE

      DO 513 M=1,49

      DPPP(M)=0.0

 513  CONTINUE

      WRITE(6,15)
 15   FORMAT(1H1,7X,5HORDER,11X,4HCOEF,/)
C     READ COEF. OF HIGHEST POWER OF S IN NUMERATOR
      READ(5,1)K,COEF
 1    FORMAT(I5,F15.0)
      WRITE(6,111)
 111  FORMAT(1H ,7X,18HTERMS IN NUMERATOR,/)
      WRITE(6,19) K,COEF
19    FORMAT(1H ,5X,I4,5X,1P2E18.7)
      III=0
      KN=K+1
      I=51-K
      KP1=K+1
      CON(KP1)=COEF
      DN=I
      DNP1=DN+1
      N(I)=COEF
C     READ REST OF COEF. OF NUMERATOR
```

```
      GO TO 60

    2 READ(5,1)K,COEF
      WRITE(6,19) K,COEF
      I=51-K
      KP1=K+1
      CON(KP1)=COEF
      N(I)=COEF
C   ZERO COEF CHECK,START COEF OF OENOMINATOR

60    IF(K.NE.0)GO TO 2

C     READ COEF OF HIGH. POWER OF S IN OEN.
      WRITE(6,112)
  112 FORMAT(1H ,//,7X,20HTERMS IN DENOMINATOR,/)
      READ(5,1)K,COEF
      WRITE(6,19) K,COEF
      KD=K+1
      I=52-K
      KP1=K+1
      COD(KP1)=COFF
C     OEGREE OF OEN.
      OD=I
      DDP1=DD+1
      D(I)=COEF
C     REST OF COEF. OF OEN.

    3 READ(5,1)K,COEF
      WRITE(6,19) K,COEF
      I=52-K
      KP1=K+1
      COD(KP1)=COFF
      D(I)=COEF
```

```
      IF(K.NE.0)GO TO 3
C   END READING OF COEF,START DERIVATIVE CALCULATIONS
C      DERIVATIVE OF DEN.
      DO 4 I=DD,51
      AI=52-I
    4 DP(I)=AI*D(I)
C      2ND DERIVATIVE OF DEN.
      DO 5 I=DD,50
      AI=51-I
    5 DPP(I)=AI*DP(I)
```

```
                                        I                                     I
                                        I                                     I
                                        I                                     I
                                        I                                     I
          ***************************************************************     I
          *C     3RD DER. OF DEN.                                        *    I
          ***************************************************************     I
                                        I                                     I
                                        I                                     I
                                        I                                     I
          ***************************************************************     I
I---------*      DO 6 I=DD,49                                            *    I
I         ***************************************************************     I
I                                       I                                     I
I                                       I                                     I
I                                       I                                     I
I         ***************************************************************     I
I         *      AI=50-I                                                 *    I
I         ***************************************************************     I
I                                       I                                     I
I                                       I                                     I
I                                       I                                     I
I         ***************************************************************     I
----------*    6 DPPP(I)=AI*DPP(I)                                       *    I
          ***************************************************************     I
                                        I                                     I
                                        I                                     I
                                        I                                     I
          ***************************************************************     I
          *C     GET DER. OF NUM.                                        *    I
          ***************************************************************     I
                                        I                                     I
                                        I                                     I
                                        I                                     I
          ***************************************************************     I
I---------*      DO 7 I=DN,50                                            *    I
I         ***************************************************************     I
I                                       I                                     I
I                                       I                                     I
I                                       I                                     I
I         ***************************************************************     I
I         *      AI=51-I                                                 *    I
I         ***************************************************************     I
I                                       I                                     I
I                                       I                                     I
I                                       I                                     I
I         ***************************************************************     I
----------*    7 NP(I)=AI*N(I)                                           *    I
          ***************************************************************     I
                                        I                                     I
                                        I                                     I
                                        I                                     I
          ***************************************************************     I
          *      WRITE(6,115)                                            *    I
          *115   FORMAT(1H ,////,8X,5HALPHA,12X,4HBETA,10X,1HK,19X,       *    I
          *     120HINVERSE LAPLACE FORM,//)                             *    I
          *C     FIRST ORDER POLE AT ORIGIN                              *    I
          ***************************************************************     I
                                        I                                     I
```

```
*       IF(D(52).EQ.0.0.AND.D(51).NE.0.0)GO TO 11

*       GO TO 10

*    11 ALPHA=0.0
*       BETA=0.0
*       K=1
*       AORB=N(51)/D(51)
*       THORC=0.0
*       WRITE(6,90)ALPHA,BETA,K,AORB,THORC
*       III=III+1
*       PALPH(III)=ALPHA
*       KK(III)=K
*       PAORB(III)=AORB
*       PBETA(III)=BETA
*       PTHRC(III)=THORC

*       GO TO 40

* 10    IF(D(52).EQ.0.0.AND.D(51).EQ.0.0.AND.D(50).NE.0.0)GOTO13

*       GOTO12

*C      SECOND ORDER POLE AT ORIGIN
```

```
*******************************************************************************
*    13 ALPHA=0.0                                                             *
*       BETA=0.0                                                              *
*       K=2                                                                   *
*       AORB=(D(50)*N(50)-D(49)*N(51))/D(50)**2                               *
*       THORC=N(51)/D(50)                                                     *
*       WRITE(6,90)ALPHA,BETA,K,AORB,THORC                                    *
*       III=III+1                                                             *
*       PALPH(III)=ALPHA                                                      *
*       KK(III)=K                                                             *
*       PAORB(III)=AORB                                                       *
*       PBETA(III)=BETA                                                       *
*       PTHRC(III)=THORC                                                      *
*******************************************************************************

*******************************************************************************
*       GO TO 40                                                              *
*******************************************************************************

*******************************************************************************
* 12    IF(D(52).EQ.0.0.AND.D(51).EQ.0.0.AND.D(50).EQ.0.0)GOTO14              *
*******************************************************************************

*******************************************************************************
*C      NO POLES AT ORIGIN - GO TO READ POLE CD.                              *
*******************************************************************************

*******************************************************************************
*       GO TO 40                                                              *
*******************************************************************************

*******************************************************************************
*C      THIRD ORDER POLE AT ORIGIN                                            *
*******************************************************************************

*******************************************************************************
* 14    ALPHA=0.0                                                             *
*       BETA=0.0                                                              *
*       K=3                                                                   *
*       AORB=(D(49)*D(49)*N(49)-D(49)*D(47)*N(51)-D(49)*D(48)*N(50)           *
*      1+N(51)*D(48)*D(48))/(D(49)*D(49)*D(49))                               *
*       THORC=(D(49)*N(50)-D(48)*N(51))/(D(49)*D(49))                         *
*       DTOP=N(51)/(2.0*D(49))                                                *
```

```
      WRITE(6,92)ALPHA,BETA,K,AORB,THORC,DTOP
      III=III+1
      PALPH(III)=ALPHA
      KK(III)=K
      PAORB(III)=AORB
      PBETA(III)=BETA
      PTHRC(III)=THORC
      PDTOP(III)=DTOP
C     POLES AT ORIGIN TESTED FOR,READ OTHER POLES

   40 READ(5,20)ALPHA,BETA,K
 20   FORMAT(2F15.0,I5)
      WRITE(6,9) ALPHA,BETA,K
    9 FORMAT(1H0,1P2E18.7,I4,2E18.7)

      IF(ALPHA.EQ.0.0.AND.BETA.EQ.0.) GO TO 109

      IF(BETA.NE.0.0)GOTO50

      IF(K.NE.2)GO TO 30

      NO=N(ON)

      DO 21 I=ONPI,51

   21 NO=-ALPHA*NO+N(I)
```

```
      IF(DNP1.GT.51)N0=N(DN)
      N1=NP(DN)
      DO 22 I=DNP1,50
   22 N1=-ALPHA*N1+NP(I)
      IF(DNP1.GT.50)N1=NP(DN)
      D2=DPP(DD)
      DO 23 I=DDP1,50
   23 D2=-ALPHA*D2+DPP(I)
      IF(DDP1.GT.50)D2=DPP(DD)
      D3=DPPP(DD)
      DO 24 I=DDP1,49
   24 D3=-ALPHA*D3+DPPP(I)
```

```
      IF(DDP1.GT.49)D3=DPPP(DD)
      AORB=(D2/2.0*N1-NO*D3/6.0)/(D2**2/4.0)
      THORC=2.0*NO/D2
      WRITE(6,93)ALPHA,BETA,K,AORB,THORC
      III=III+1
      PALPH(III)=ALPHA
      KK(III)=K
      PAORB(III)=AORB
      PBETA(III)=BETA
      PTHRC(III)=THORC

      GO TO 40                                  --->A

O<---------------------------------------------------O

   30 IF(BETA.NE.0.0)GO TO 50                   --->V

      NO=N(DN)

      DO 31 I=DNP1,51

   31 NO=-ALPHA*NO+N(I)

      IF(DNP1.GT.51)NO=N(DN)
      D1=CP(DD)
```

```
      DO 32 I=DDP1,51

   32 D1=-ALPHA*D1+DP(I)

      IF(DDP1.GT.51)D1=DP(DD)
      AORB=NO/D1
      THORC=0.0
      WRITE(6,91)ALPHA,BETA,K,AORB,THORC
      III=III+1
      PALPH(III)=ALPHA
      KK(III)=K
      PAORB(III)=AORB
      PBETA(III)=BETA
      PTHRC(III)=THORC

      GO TO 40

C     COMPLEX POLES

   50 TOTNR=CON(1)
      TOTNI=0.
      TOTDR=COD(2)
      TOTDI=0.0
      A=-ALPHA
      B=BETA
      AX=1.0
      AY=0.0
```

```
      DO 300 IR=1,KD

      RE(IR)=A*AX-B*AY
      IM(IR)=B*AX+A*AY
      AX=RE(IR)
      AY=IM(IR)

  300 CONTINUE

      DO 301 IN=1,KN

      INP1=IN+1

      IF(INP1. GT.KN)GO TO 305

      TOTNR=CON(IN+1)*RE(IN)+TOTNR

  301 TOTNI=CON(IN+1)*IM(IN)+TOTNI
```

```
 305  CONTINUE

      DO 302 ID=1,KD

      DIF=ID+1
      IDP2=ID+2

      IF(IDP2.GT.KD)GO TO 306

      TDTDR=CDD(ID+2)*RE(ID)*DIF+TDTDR

 302  TOTDI=CDD(ID+2)*IM(ID)*DIF+TOTDI

 306  CONTINUE
      AMD=SQRT(TDTDR**2+(TDTDI**2))
      AMN=SQRT(TDTNR**2+(TOTNI**2))
      ANGD=ATAN2(TDTDR,TOTDI)
      ANGN=ATAN2(TDTNR,TDTNI)
      AORB=2.0*AMN/AMD
      THDRC=ANGN-ANGD
      YIM=AORB*SIN(THDRC)
      YRE= AORB*COS(THORC)
      THORC=ATAN2(YRE,YIM)
C     IF IN FIRST DR FDURTH QUAD,LEAVE IT
C     OTHER WISE ADD DR SUB PI TO GET SUCH, AND REVERSE
C     SIGN TO COMPENSATE
```

```
*      IF(ABS(THORC).LE.1.57079632) GO TO 453

*      IF(THORC)450,451,452

*450   THORC=THORC+3.14159265

*      GOTO 451

*452   THORC=THORC-3.1415926

*451   AORB=-AORB

* 453  CONTINUE
*      WRITE(6,95)ALPHA,BETA,K,AORB,THORC
*      III=III+1
*      PALPH(III)=ALPHA
*      KK(III)=K
*      PAORB(III)=AORB
*      PBETA(III)=BETA
*      PTHRC(III)=THORC
```

```
      GO TO 40
 10)  CONTINUE
      DO 1100 ITT=1,15
      CALL PLOT(PALPH,PBETA,KK,PAORB,PTHRC,TSTA,TSTOP,ITT,PDTOP,
     1YSTA,YSTOP)
      IF(TSTA(ITT+1).EQ.0.0.AND.TSTOP(ITT+1).EQ.0.0)GO TO 1200
1100  CONTINUE
1200  CONTINUE
      ICSE=ICSE+1
      GOTO999
      END
```

```
                                   (ENTRANCE)
                                       I
                                       I
         ***********************************************************************
         *C   END COMPUTATIONS,START GRAPHICAL DISPLAY                          *
         *C     PROGRAM TO CHOOSE TIME INTERVAL AND GRAPH RESULTS               *
         *      SUBROUTINE PLOT(PALPH,PBETA,KK,PAORB,PTHRC,TSTA,TSTOP,          *
         *     1ITT,PDTOP,YSTA,YSTOP)                                           *
         *      REAL TSTA(15),TSTOP(15),YSTA(15),YSTOP(15)                      *
         *      INTEGER  ITT                                                    *
         *      REAL VALT(15)                                                   *
         *      REAL  FT(15),TIME(15)                                           *
         *      REAL PALPH(60),PBETA(60),PAORB(60),PTHRC(60)                    *
         *      REAL  PDTOP(60)                                                 *
         *      INTEGER KK(60),K(60),I,L,J                                      *
         *      REAL ALPHA(60),BETA(60),AORB(60),THORC(60),E,F,T,TF,FP          *
         *      REAL ALPHAS,ALPHAL,BETAS,BETAL,DELTAT,TEND,DELTA                *
         *      INTEGER IS                                                      *
         *      REAL  GRAPH(60,130)                                             *
         *      DATA BLANK/1H /, DOT/1H./, X/1HX/                               *
         *      TEND=TSTOP(ITT)                                                 *
         *      BPO=-99999999999.                                               *
         *      SNE=99999999999.                                                *
         *      ZAP=SNE                                                         *
         *      AI=0.                                                           *
         *      BJ=0.                                                           *
         *      FI=0.0                                                          *
         *      FJ=C.0                                                          *
         ***********************************************************************
                                       I
                                       I
                                       I
         ***********************************************************************
I--------*      DO 430 NCL=1,15                                                 *
I        ***********************************************************************
I                                      I
I                                      I
I                                      I
I        ***********************************************************************
I        *      FT(NCL)=0.0                                                     *
I        *      TIME(NCL)=0.C                                                   *
I        ***********************************************************************
I                                      I
I                                      I
I                                      I
I        ***********************************************************************
---------* 430  VALT(NCL)=0.0                                                   *
         ***********************************************************************
                                       I
                                       I
                                       I
         ***********************************************************************
         *      FT(1)=1.0                                                       *
         *      FT(2)=2.0                                                       *
         *      FT(3)=3.0                                                       *
         *      FT(4)=4.0                                                       *
         *      FT(5)=5.0                                                       *
         *      VALT(I)=TSTA(ITT)                                               *
         *      GEST=0.0                                                        *
         *      BEST=0.0                                                        *
```

```
*       TEST=0.0                                                                   *
*C      ARRAY FOR GRAPH                                                            *
************************************************************************************
                                   I
```

```
      DO 612 I=1,60
      DO 612 J=1,130
612   GRAPH(I,J)=BLANK
C     DOTS FOR AXES
      DO 667 I=10,60
667   GRAPH(I,10)=DOT
      DO 670 I=10,12
      DO 670 J=10,130,10
670   GRAPH(I,J)=DOT
```

```
      DO 668 J=10,130
 668  GRAPH(10,J)=DOT
C     WRITE HEADING
      WRITE(6,9998)
9998  FORMAT(1H1,15X,1HT,13X,3HFOT,10X,5HDELTA,7X,6HDELTAT,//)
      DO 501 L=1,60
      M=L
      ALPHA(M)=PALPH(M)
      BETA(M)=PBETA(M)
      K(M)=KK(M)
      AORB(M)=PAORB(M)
      THORC(M)=PTHRC(M)
      IF(K(M).EQ.0) GO TO 64
 501  CONTINUE
64    L=L-1
C     GET ALPHAS,ALPHAL,IS,BETAS,BETAL
      IS=1
      ALPHAS=ABS(ALPHA(1))
      ALPHAL=ABS(ALPHA(1))
```

```
*       BETAS=ABS(BETA(1))                                                    *
*       BETAL=ABS(BETA(1))                                                    *
***************************************************************************
                                     I
```

```
                                        I
                                        I
                                        I
                                        I
         ********************************************************************
I--------*      DO 23 I=2,L                                                 *
I        ********************************************************************
I                                       I
I                                       I
I                                       I
I        ********************************************************************
I        *      IF(ALPHAS.LT.ABS(ALPHA(I)))GO TO 24                         *------0
I        ********************************************************************      I
I                                       I                                          I
I                                       I                                          I
I                                       I                                          I
I        ********************************************************************      I
I        *      ALPHAS=ABS(ALPHA(I))                                        *      I
I        *      IS=I                                                        *      I
I        ********************************************************************      I
I                                       I                                          I
I                                       0<-----------------------------------------0
I                                       I
I        ********************************************************************
I        *24    ALPHAL=AMAX1(ALPHAL,ABS(ALPHA(I)))                          *
I        ********************************************************************
I                                       I
I                                       I
I                                       I
I        ********************************************************************
I        *      IF(BETA(I).EQ.0.0)GO TO 23                                  *------0
I        ********************************************************************      I
I                                       I                                          I
I                                       I                                          I
I                                       I                                          I
I        ********************************************************************      I
I        *      BETAS=AMIN1(BETAS,ABS(BETA(I)))                             *      I
I        *      BETAL=AMAX1(BETAL,ABS(BETA(I)))                             *      I
I        ********************************************************************      I
I                                       I                                          I
I                                       0<-----------------------------------------0
I                                       I
I        ********************************************************************
---------*23    CONTINUE                                                    *
         ********************************************************************
                                        I
                                        0<-----------------------------------------0
                                        I                                          I
         ********************************************************************      I
         * 327  CONTINUE                                                    *      I
         *C   FIND INITIAL STEP SIZE,DELTAT                                 *      I
         ********************************************************************      I
                                        I                                          I
                                        I                                          I
```

```
*       IF(ALPHAL.GT.BETAL)GO TO 25

*       OELTAT=.I/BETAL

*       GO TO 26

* 25    OELTAT=.1/ALPHAL

* 26    CONTINUE
*       T=TSTA(ITT)
*C      EVALUATE FUNCTION

* 67    F=0.0

*       DO 78 I=I,L

*       IF(BETA(I))80,81,80

*C      COMPLEX-NOT ZERO BETA
```

```
*80     F=F+AORB(I)*EXP(-ALPHA(I)*T)*SIN(BETA(I)*T+THORC(I))

*       GO TO 78

*C      REAL - BETA ZERO

*81     E=EXP(-ALPHA(I)*T)
*       F=F+AORB(I)*E
*       IF(K(I).EQ.2)F=F+THORC(I)*T*E
*       IF(K(I).EQ.3)F=F+PDTOP(I)*T*T*E

*78     CONTINUE

*       IF(YSTA(ITT).NE.0.0.AND.YSTOP(ITT).NE.0.0)GOTO326

*       IF(TEST.EQ.1.0) GO TO 326

*       DO 411 NTE=1,4
```

```
      FT(NTE)=FT(NTE +1)
      TIME(NTE)=T(ME(NTE + 1)

 411  CONTINUE

      FT(5)=F
      TIME(5)=T

      IF(T.GT.TSTOP(ITT))GO TO 333

C     DETERMINE SCALING OF AXIS
      SNE=AMIN1(SNE,F)
      BPO=AMAXI(BPO,F)

      DO 412 NTE=2,5

      DJAA=ABS(FT(NTE-1)-FT(NTE))

      IF(DJAA.GT.(ABS(FT(I))*.00000001))GOTO 49
```

```
  412  CONTINUE

       TEND=TIME(1)
       TEST=1.0

       GO TO 327                                        ----->A

  333  TEND=TSTOP(ITT)
       TEST=1.0

       GO TO 327

  326  CONTINUE

       IF(BEST.EQ.1.0) GO TO 399

       IF(YSTA(ITT).EQ.0.0.AND.YSTOP(ITT).EQ.0.0)GOTO305

       BPG=YSTOP(ITT)
       SNE=YSTA(ITT)
```

```
      305  CONTINUE
           FI=50.0/(BPO-SNE)
           FJ=120.0/(TEND-TSTA(ITT))
           BEST=1.0

      399  CONTINUE
           TF=F
           WRITE(6,72)T,TF,DELTA,DELTAT
      72   FORMAT(1H ,4X,2F15.4,6X,2F9.4)
           AI=(FI*(TF-SNE))+10.0

           IF(AI.GT.100000.)GOTO450

           I=IFIX(AI)

           IF(GEST.EQ.1.0)GOTO39

           ZPT=BPO
           SZRO=(BPO-SNE)/50.0

           DO 350 KT=1,50
```

```
      ZAP=AMIN1(ZAP,ABS(ZPT))
      ZPT=ZPT-SZRO

 350  CONTINUE

      ZI=(FI*(ZAP-SNE))+10.0
      JA=IFIX(ZI)

      DO 92 J=10,130,5

 92   GRAPH(JA,J)=OOT

      GEST=1

 39   CONTINUE
      BJ=(FJ*(T-TSTA(ITT)))+10.0

      IF(BJ.GT.100000.00)GOTO 450

      J=IFIX(BJ)
      IF(TF.EQ.SNF) I=10
      IF(TF.EQ.BPO)  I=60
      IF(T.EQ.TSTA(ITT))  J=10
      IF(T.EQ.TEND)  J=130
```

```
      IF(I.GE.10.AND.I.LE.60.AND.J.GE.10.AND.J.LE.130)GO TO 400
      GO TO 450
  400 GRAPH(I,J)=X
  450 CONTINUE
      IF(T.GE.TEND)GO TO 666
C     FIND DERIVATIVE AT T=T AND T=T+DELTAT
 49   FP=0.0
      DO 178 I=1,L
      E=EXP(-ALPHA(I)*T)
```

```
*       IF(BETA(I))180,181,180

*C      COMPLEX - BETA NOT ZERO

*180    FP=FP+AORB(I)*E*COS(BETA(I)*T+THORC(I))*BETA(I)
*      1 -AORB(I)*ALPHA(I)*E*SIN(BETA(I)*T+THORC(I))

*       GO TO 178

*C      REAL - BETA ZERO

*181    FP=FP-ALPHA(I)*AORB(I)*E
*       IF(K(I).EQ.2)FP=FP+THORC(I)*E*(1.0-ALPHA(I)*T)
*       IF(K(I).EQ.3)FP=FP+PDTOP(I)*E*(2.0*T-ALPHA(I)*T*T)

*178    CONTINUE

*       AX=2.0*FP
*       Y=T+DELTAT
*       FP=0.0
```

```
      DO 278 I=1,L

      E=EXP(-ALPHA(I)*Y)

      IF(BETA(I))280,281,280

280   FP=FP+AORB(I)*E*COS(BETA(I)*Y+THORC(I))*BETA(I)
     1 -AORB(I)*ALPHA(I)*E*SIN(BETA(I)*Y+THORC(I))

      GO TO 278

281   FP=FP-ALPHA(I)*AORB(I)*E
      IF(K(I).EQ.2)FP=FP+THORC(I)*E*(1.0-ALPHA(I)*Y)
      IF(K(I).EQ.3)FP=FP+PDTOP(I)*E*(2.0*Y-ALPHA(I)*Y*Y)

278   CONTINUE

      FP=2.0*FP
      DELTA=ABS(ATAN(AX)-ATAN(FP))
C     END DERV. EVAL.
```

```
                                                 I                                  I    I    I
                                                 I                                  I    I    I
                                                 I                                  I    I    I
                                                 I                                  I    I    I
**************************************************************************          I    I    I
*       IF(DELTA.GE.D.05)GO TO 50                                        *------I---I---I-------0
**************************************************************************          I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
**************************************************************************          I    I    I        I
*       T=T+DELTAT                                                       *          I    I    I        I
*       IF(T.GE.TENO)T=TENO                                              *          I    I    I        I
*       DELTAT=1.500*OELTAT                                              *          I    I    I        I
**************************************************************************          I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
**************************************************************************          I    I    I        I
*       GD TO 67                                                         *------I---I-->A        I
**************************************************************************          I    I    I        I
                                                 I                                  I    I    I        I
                                                0<------------------------------------I---I---I-------D
                                                 I                                  I    I    I
**************************************************************************          I    I    I
* 50    IF(DELTA.GE.D.15.AND.OELTAT.GE.ITSTOP(ITT)-TSTA(ITT))/           *------I---I---I-------0
*      1120.0)GO TD 60                                                   *          I    I    I        I
**************************************************************************          I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
**************************************************************************          I    I    I        I
*       T=T+DELTAT                                                       *          I    I    I        I
*       IF(T.GE.TENO)T=TEND                                              *          I    I    I        I
**************************************************************************          I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
                                                 I                                  I    I    I        I
**************************************************************************          I    I    I        I
*       GO TO 67                                                         *------I---I---0        I
**************************************************************************          I    I             I
                                                 I                                  I    I             I
                                                0<------------------------------------I---I-----------D
                                                 I                                  I    I
**************************************************************************          I    I
*60     DELTAT=DELTAT/2.D                                                *          I    I
**************************************************************************          I    I
                                                 I                                  I    I
                                                 I                                  I    I
                                                 I                                  I    I
**************************************************************************          I    I
*       GD TD 49                                                         *------I---0
**************************************************************************          I
                                                                                    I
```

```
 666  WRITE(6,120)TSTA((TT),TENO,TSTOP((TT)
 120  FORMAT(1H1,11HSTART TIME=,F10.4,10X,17HACTUAL  STOP  TIME=,F10.4,
     110X,19HPROPOSED STOP TIME=,F10.4)
      WR(TE(6,8)
 8    FORMAT(1H ,4X,4HF(T),/)
      VALY=BPO
      SCAY=(BPO-SNE)/50.0

      00 29 I1=1,51

      I=61-I1
      WRITE(6,30)VALY,(GRAPH(I,J),J=10,130)
      VALY=VALY-SCAY

 29   CONTINUE

 30   FORMAT(1H ,F8.3,1X,121A1)
      SCAX=(TEND-VALT(1))/12.00

      00 68C LAX=2,13

 680  VALT(LAX)=VALT(LAX-1)+SCAX
```

```
                                    I
                                    I
                                    I
*       WRITE(6,5)(VALT(LAX),LAX=1,13)
* 5     FORMAT(1H ,2X,12F10.3,F9.3)
*       WRITE(6,7)
* 7     FORMAT(1H ,/,50X,14HREAL TIME(SEC))
*C      TAKE ANOTHER SET OF OATA
                                    I
                                    I
                                    I
*       RETURN

*       END
```

TABLE 4. EXAMPLE 1 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 2, G(s) = 1/s

```
ORDER            COEF

TERMS IN NUMERATOR

 1          3.0000000E 00
 0          1.1000000E 01

TERMS IN DENOMINATOR

 3          1.0000000E 00
 2          8.0000000E 00
 1          1.5000000E 01
 0          0.0000000E 00


  ALPHA            BETA           K                INVERSE LAPLACE FORM

0.0000000E 00    0.0000000E 00    1     (     7.3333333E-01+(      0.0000000E 00)*T

5.0000000E 00    0.0000000E 00    1
5.0000000E 00    0.0000000E 00    1     (    -4.0000000E-01)*EXP(-ALPHA*T)
0.0000000E 00

3.0000000E 00    0.0000000E 00    1
3.0000000E 00    0.0000000E 00    1     (    -3.3333333E-01)*EXP(-ALPHA*T)
0.0000000E 00

0.0000000E 00    0.0000000E 00    0
```

TABLE 5. EXAMPLE 1 - TABULATION OF RESPONSE VERSUS TIME (0 TO 2.5 SECONDS) FOR FIGURE 2

T	FOT	DELTA	OELTAT
.0000	.0000	.0011	.0200
.0200	.0575	.0146	.0400
.0600	.1586	.0328	.0800
.1400	.3157	.0817	.0800
.2200	.4279	.1058	.0800
.3000	.5086	.1303	.0800
.3800	.5669	.1492	.0800
.4200	.5898	.0781	.0400
.4600	.6094	.0780	.0400
.5000	.6261	.0761	.0400
.5400	.6405	.0725	.0400
.5800	.6528	.0677	.0400
.6200	.6634	.0621	.0400
.6600	.6726	.0561	.0400
.7000	.6804	.0502	.0400
.7400	.6872	.0444	.0800
.8200	.6982	.0733	.0800
.9000	.7065	.0559	.0800
.9800	.7127	.0423	.1600
1.1400	.7211	.0561	.1600
1.3000	.7260	.0322	.3200
1.6200	.7306	.0298	.6400
2.2600	.7329	.0144	1.2800
2.5000	.7331	.0023	2.5600

TABLE 6. EXAMPLE 1 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 3,
$G(s) = 1/(s + 1)$

ORDER	COEF
TERMS IN NUMERATOR	
1	3.0000000E 00
0	1.1000000E 01
TERMS IN DENOMINATOR	
3	1.0000000E 00
2	9.0000000E 00
1	2.3000000E 01
0	1.5000000E 01

ALPHA	BETA	K	INVERSE LAPLACE FORM
3.0000000E 00	0.0000000E 00	1	
3.0000000E 00	0.0000000E 00	1	(-5.0000000E-01)*EXP(-ALPHA*T)
0.0000000E 00			
1.0000000E 00	0.0000000E 00	1	
1.0000000E 00	0.0000000E 00	1	(1.0000000E 00)*EXP(-ALPHA*T)
0.0000000E 00			
5.0000000E 00	0.0000000E 00	1	
5.0000000E 00	0.0000000E 00	1	(-5.0000000E-01)*EXP(-ALPHA*T)
0.0000000E 00			
0.0000000E 00	0.0000000E 00	0	

TABLE 7. EXAMPLE 1 - TABULATION OF RESPONSE VERSUS TIME
(0 TO 5.0 SECONDS) FOR FIGURE 3

T	FOT	DELTA	DELTAT
.0000	.0000	.0135	.0200
.0200	.0569	.0183	.0300
.0500	.1315	.0318	.0450
.0950	.2224	.0596	.0450
.1400	.2925	.0780	.0450
.1850	.3456	.1024	.0450
.2300	.3854	.1331	.0450
.2525	.4010	.0796	.0225
.2750	.4140	.0884	.0225
.2975	.4249	.0966	.0225
.3200	.4338	.1032	.0225
.3425	.4408	.1074	.0225
.3650	.4463	.1085	.0225
.3875	.4504	.1064	.0225
.4100	.4531	.1014	.0225
.4325	.4548	.0941	.0225
.4550	.4554	.0855	.0225
.4775	.4550	.0764	.0225
.5000	.4539	.0673	.0225
.5225	.4521	.0588	.0225
.5450	.4496	.0510	.0225
.5675	.4465	.0440	.0338
.6013	.4410	.0547	.0338
.6350	.4346	.0434	.0506
.6856	.4236	.0483	.0759
.7616	.4049	.0449	.1139
.8755	.3742	.0280	.1709
1.0463	.3269	.0060	.2563
1.3026	.2610	.0553	.2563
1.5589	.2055	.0717	.2563
1.8152	.1606	.0700	.2563
2.0715	.1250	.0615	.2563
2.3278	.0970	.0512	.2563
2.5841	.0753	.0414	.3844
2.9685	.0513	.0465	.5767
3.5451	.0289	.0444	.8650
4.4101	.0122	.0333	1.2975
5.0000	.0067	.0177	1.9462

TABLE 8. EXAMPLE 1 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 4,
$G(s) = (1 + 0.5s)/(1 + 0.3s + 0.25s^2)$

ORDER	COEF
TERMS IN NUMERATOR	
2	1.5000000E 00
1	8.5000000E 00
0	1.1000000E 01
TERMS IN DENOMINATOR	
4	2.5000000E-01
3	2.3000000E 00
2	7.1500000E 00
1	1.2500000E 01
0	1.5000000E 01

ALPHA	BETA	K	INVERSE LAPLACE FORM
6.0000000E-01	1.9078784E 00	1	
6.0000000E-01	1.9078784E 00	1	(1.8280140E 00)*EXP(-ALPHA*T)*SIN((BETA*T)+(4.1348706E-01))
3.0000000E 00	0.0000000E 00	1	
3.0000000E 00	0.0000000E 00	1	(-2.1276596E-01)*EXP(-ALPHA*T)
0.0000000E 00			
5.0000000E 00	0.0000000E 00	1	
5.0000000E 00	0.0000000E 00	1	(-5.2173913E-01)*EXP(-ALPHA*T)
0.0000000E 00			
0.0000000E 00	0.0000000E 00	0	

TABLE 9. EXAMPLE 1 - TABULATION OF RESPONSE VERSUS TIME (0 TO 5.0 SECONDS) FOR FIGURE 4

T	FCT	DELTA	DELTAT
.0000	.0000	.1109	.0200
.0200	.1158	.0061	.0300
.0500	.2748	.0101	.0450
.0950	.4830	.0178	.0675
.1625	.7356	.0350	.1012
.2637	1.0024	.0865	.1012
.3144	1.0933	.0739	.0506
.3650	1.1599	.1172	.0506
.3903	1.1849	.0869	.0253
.4156	1.2046	.1175	.0253
.4409	1.2194	.1617	.0253
.4663	1.2295	.2213	.0253
.4916	1.2351	.2848	.0253
.5169	1.2365	.3182	.0253
.5422	1.2339	.2936	.0253
.5675	1.2275	.2321	.0253
.5928	1.2176	.1703	.0253
.6181	1.2042	.1233	.0253
.6434	1.1877	.0905	.0253
.6688	1.1682	.0680	.0253
.6941	1.1459	.0523	.0253
.7194	1.1209	.0410	.0380
.7573	1.0790	.0467	.0570
.8143	1.0072	.0481	.0854
.8997	.8830	.0432	.1281
1.0279	.6719	.0297	.1922
1.2201	.3360	.0019	.2883
1.5084	-.1129	.0803	.2883
1.6526	-.2811	.0993	.1442
1.7247	-.3476	.0775	.0721
1.7967	-.4017	.1066	.0721
1.8688	-.4433	.1474	.0721
1.9049	-.4596	.0932	.0360
1.9409	-.4728	.1078	.0360
1.9769	-.4831	.1223	.0360
2.0130	-.4904	.1349	.0360
2.0490	-.4950	.1433	.0360
2.0851	-.4969	.1454	.0360
2.1211	-.4962	.1408	.0360
2.1571	-.4930	.1304	.0360
2.1932	-.4874	.1166	.0360
2.2292	-.4796	.1015	.0360
2.2653	-.4697	.0867	.0360
2.3013	-.4578	.0733	.0360
2.3373	-.4441	.0616	.0360
2.3734	-.4287	.0515	.0360
2.4094	-.4118	.0429	.0541
2.4635	-.3840	.0510	.0541
2.5176	-.3535	.0381	.0811
2.5986	-.3040	.0382	.1216
2.7203	-.2247	.0250	.1825
2.9027	-.1043	.0183	.2737
3.1764	.0515	.1447	.2737
3.3133	.1093	.1336	.1368
3.3817	.1321	.0838	.0684
3.4501	.1508	.0943	.0684
3.5185	.1653	.1030	.0684

TABLE 9 (Continued)

3.5869	.1757	.1084	.0684
3.6554	.1821	.1095	.0684
3.7238	.1848	.1062	.0684
3.7922	.1839	.0988	.0684
3.8606	.1798	.0885	.0684
3.9290	.1728	.0766	.0684
3.9975	.1632	.0641	.0684
4.0659	.1515	.0520	.0684
4.1343	.1379	.0405	.1026
4.2369	.1151	.0412	.1539
4.3909	.0777	.0237	.2309
4.6218	.0220	.0347	.3464
4.7950	-.0141	.0698	.1732
4.9682	-.0418	.0965	.1732
5.0000	-.0459	.1100	.1732

TABLE 10. EXAMPLE 2 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 5,
$G(s) = 1/s$

```
OROER              COEF

TERMS IN NUMERATOR

 2           1.3400000E-03
 1           2.0670000E-01
 0           1.0000000E 00

TERMS IN DENOMINATOR

 6           6.9981000E-07
 5           1.1552908E-04
 4           1.6800000E-03
 3           3.8280000E-02
 2           2.5930000E-01
 1           1.0000000E 00
 0           0.0000000E 00


  ALPHA             BETA          K                         INVERSE LAPLACE FORM

0.0000000E 00    0.0000000E 00   1    (     1.0000000E 00+(      0.0000000E 00)*T

4.1258865E 00    4.2939554E 00   1
4.1258865E 00    4.2939554E 00   1    (     1.3297239E 00)*EXP(-ALPHA*T)*SIN((8ETA*T)+(     -9.1383562E-01))

2.6595745E 00    1.6089876E 01   1
2.6595745E 00    1.6089876E 01   1    (    -4.8119085E-01)*EXP(-ALPHA*T)*SIN((8ETA*T)+(     -1.1187349E-01))

1.5151510E 02    0.0000000E 00   1
1.5151510E 02    0.0000000E 00   1    (    -8.5925566E-C6)*EXP(-ALPHA*T)
0.0000000E 00

0.0000000E 00    0.0000000E 00   0
```

TABLE 11. EXAMPLE 2 - TABULATION OF RESPONSE VERSUS TIME (0 TO 5.0 SECONDS) FOR FIGURE 5

T	FOT	OELTA	OELTAT
.0000	.0008	.0000	.0007
.0007	.0008	.0007	.0010
.0017	.0008	.0042	.0015
.0031	.0008	.0132	.0022
.0054	.0008	.0350	.0033
.0087	.0009	.0853	.0033
.0120	.0013	.1202	.0033
.0154	.0018	.1454	.0033
.0187	.0027	.1559	.0033
.0221	.0039	.1510	.0033
.0254	.0055	.1350	.0033
.0288	.0076	.1143	.0033
.0321	.0102	.0938	.0033
.0354	.0134	.0759	.0033
.0388	.0172	.0613	.0033
.0421	.0216	.0496	.0050
.0471	.0295	.0579	.0050
.0521	.0390	.0435	.0075
.0597	.0566	.0470	.0113
.0709	.0907	.0455	.0169
.0878	.1597	.0386	.0254
.1132	.3017	.0279	.0381
.1513	.5788	.0144	.0571
.2084	1.0166	.0099	.0856
.2512	1.2523	.0507	.0428
.2726	1.3207	.0839	.0214
.2940	1.3542	.3581	.0214
.3154	1.3558	1.8784	.0214
.3368	1.3309	.4069	.0214
.3582	1.2868	.0831	.0214
.3796	1.2319	.0236	.0321
.4117	1.1467	.0140	.0482
.4358	1.0927	.0536	.0241
.4599	1.0545	.1423	.0241
.4840	1.0351	.4912	.0241
.5081	1.0341	1.2352	.0241
.5322	1.0484	.4851	.0241
.5563	1.0727	.1214	.0241
.5803	1.1010	.0115	.0361
.6165	1.1380	.1532	.0361
.6526	1.1552	.7784	.0361
.6887	1.1461	1.1192	.0361
.7248	1.1136	.2752	.0361
.7610	1.0676	.0555	.0361
.7971	1.0209	.0412	.0542
.8242	.9923	.1096	.0271
.8513	.9729	.2495	.0271
.8784	.9637	.5207	.0271
.9055	.9642	.6250	.0271
.9326	.9721	.3367	.0271
.9597	.9845	.1122	.0271
.9868	.9979	.0198	.0406
1.0274	1.0137	.2538	.0406
1.0680	1.0193	.5209	.0406
1.1087	1.0138	.4409	.0406
1.1493	1.0013	.1521	.0406
1.1900	.9877	.0707	.0406

TABLE 11 (Continued)

1.2306	.9786	.2635	.0406
1.2713	.9768	.3650	.0406
1.3119	.9818	.2639	.0406
1.3525	.9907	.0844	.0406
1.3932	.9997	.0703	.0406
1.4338	1.0057	.1840	.0406
1.4745	1.0072	.2166	.0406
1.5151	1.0049	.1504	.0406
1.5557	1.0006	.0410	.0610
1.6167	.9950	.1124	.0610
1.6472	.9941	.0942	.0305
1.6777	.9945	.0867	.0305
1.7082	.9961	.0595	.0305
1.7386	.9984	.0230	.0457
1.7844	1.0017	.0311	.0686
1.8529	1.0039	.1252	.0686
1.9215	1.0021	.0846	.0686
1.9901	.9992	.0219	.1029
2.0930	.9987	.1028	.1029
2.1959	1.0010	.0090	.1543
2.3502	1.0000	.0598	.1543
2.5045	.9996	.0482	.2315
2.7360	1.0000	.0282	.3472
3.0832	1.0001	.0072	.5208
3.6040	1.0000	.0034	.7812
4.3852	1.0000	.0001	1.1718
5.0000	1.0000	.0000	1.7577

TABLE 12. EXAMPLE 2 - TABULATION OF RESPONSE VERSUS TIME (0 to 1.2 SECONDS) FOR FIGURE 6

T	FOT	OELTA	OELTAT
.0000	.0008	.1268	.0007
.0007	.0008	.0007	.0010
.0017	.0008	.0042	.0015
.0031	.0008	.0132	.0022
.0054	.0008	.0350	.0033
.0087	.0009	.0853	.0033
.0120	.0013	.1202	.0033
.0154	.0018	.1454	.0033
.0187	.0027	.1559	.0033
.0221	.0039	.1510	.0033
.0254	.0055	.1350	.0033
.0288	.0076	.1143	.0033
.0321	.0102	.0938	.0033
.0354	.0134	.0759	.0033
.0388	.0172	.0613	.0033
.0421	.0216	.0496	.0050
.0471	.0295	.0579	.0050
.0521	.0390	.0435	.0075
.0597	.0566	.0470	.0113
.0709	.0907	.0455	.0169
.0878	.1597	.0386	.0254
.1132	.3017	.0279	.0381
.1513	.5788	.0144	.0571
.2084	1.0166	.0099	.0856
.2512	1.2523	.0507	.0428
.2726	1.3207	.0839	.0214
.2833	1.3417	.1019	.0107
.2887	1.3490	.0943	.0054
.2940	1.3542	.1619	.0054
.2994	1.3574	.3092	.0054
.3047	1.3587	.5715	.0054
.3101	1.3581	.6289	.0054
.3154	1.3558	.3687	.0054
.3208	1.3518	.1893	.0054
.3261	1.3462	.1068	.0054
.3315	1.3392	.0664	.0054
.3368	1.3309	.0444	.0080
.3448	1.3162	.0434	.0120
.3569	1.2900	.0369	.0181
.3750	1.2444	.0240	.0271
.4020	1.1716	.0018	.0406
.4427	1.0800	.0905	.0406
.4630	1.0509	.1476	.0203
.4681	1.0457	.0659	.0051
.4732	1.0414	.0875	.0051
.4783	1.0380	.1183	.0051
.4833	1.0354	.1618	.0051
.4884	1.0336	.2179	.0051
.4935	1.0326	.2740	.0051
.4986	1.0325	.2998	.0051
.5037	1.0330	.2748	.0051
.5087	1.0343	.2186	.0051
.5138	1.0363	.1619	.0051
.5189	1.0389	.1177	.0051
.5240	1.0421	.0861	.0051
.5291	1.0458	.0639	.0051
.5341	1.0501	.0480	.0076

TABLE 12 (Continued)

.5418	1.0572	.0512	.0076
.5494	1.0651	.0337	.0114
.5608	1.0779	.0280	.0171
.5780	1.0982	.0054	.0257
.6037	1.1265	.0695	.0257
.6165	1.1380	.0866	.0129
.6230	1.1428	.0655	.0064
.6294	1.1469	.0874	.0064
.6358	1.1503	.1172	.0064
.6422	1.1528	.1568	.0064
.6487	1.1546	.2042	.0064
.6551	1.1554	.2481	.0064
.6615	1.1554	.2671	.0064
.6680	1.1546	.2483	.0064
.6744	1.1529	.2046	.0064
.6808	1.1503	.1574	.0064
.6873	1.1470	.1181	.0064
.6937	1.1428	.0885	.0064
.7001	1.1380	.0670	.0064
.7065	1.1325	.0513	.0064
.7130	1.1263	.0396	.0096
.7226	1.1161	.0434	.0145
.7371	1.0989	.0400	.0217
.7588	1.0705	.0230	.0326
.7913	1.0279	.0278	.0488
.8157	1.0003	.0782	.0244
.8280	.9890	.0682	.0122
.8402	.9796	.0985	.0122
.8524	.9723	.1420	.0122
.8585	.9694	.0924	.0061
.8646	.9671	.1089	.0061
.8707	.9653	.1256	.0061
.8768	.9640	.1404	.0061
.8829	.9632	.1501	.0061
.8890	.9628	.1521	.0061
.8951	.9630	.1458	.0061
.9012	.9635	.1327	.0061
.9073	.9645	.1158	.0061
.9134	.9659	.0979	.0061
.9195	.9675	.0810	.0061
.9256	.9695	.0659	.0061
.9317	.9718	.0527	.0061
.9378	.9743	.0415	.0092
.9470	.9784	.0444	.0137
.9607	.9850	.0341	.0206
.9813	.9953	.0081	.0309
1.0122	1.0088	.1374	.0309
1.0276	1.0138	.1320	.0154
1.0354	1.0157	.0830	.0077
1.0431	1.0173	.0934	.0077
1.0508	1.0184	.1019	.0077
1.0585	1.0190	.1071	.0077
1.0663	1.0193	.1084	.0077
1.0740	1.0191	.1053	.0077
1.0817	1.0186	.0985	.0077
1.0894	1.0176	.0890	.0077
1.0972	1.0163	.0779	.0077
1.1049	1.0147	.0663	.0077
1.1126	1.0128	.0549	.0077
1.1203	1.0107	.0441	.0116

TABLE 12 (Continued)

1.1319	1.0072	.0475	.0174
1.1493	1.0013	.0340	.0261
1.1754	.9923	.0222	.0391
1.1949	.9863	.0706	.0196
1.2000	.9849	.1157	.0196

TABLE 13. EXAMPLE 2 - TABULATION OF RESPONSE VERSUS TIME (0 TO 0.5 SECOND) FOR FIGURE 7

T	FOT	OELTA	OELTAT
.0000	.0008	.1534	.0007
.0007	.0008	.0007	.0010
.0017	.0008	.0042	.0015
.0031	.0008	.0132	.0022
.0054	.0008	.0350	.0033
.0087	.0009	.0853	.0033
.0120	.0013	.1202	.0033
.0154	.0018	.1454	.0033
.0187	.0027	.1559	.0033
.0221	.0039	.1510	.0033
.0254	.0055	.1350	.0033
.0288	.0076	.1143	.0033
.0321	.0102	.0938	.0033
.0354	.0134	.0759	.0033
.0388	.0172	.0613	.0033
.0421	.0216	.0496	.0050
.0471	.0295	.0579	.0050
.0521	.0390	.0435	.0075
.0597	.0566	.0470	.0113
.0709	.0907	.0455	.0169
.0878	.1597	.0386	.0254
.1132	.3017	.0279	.0381
.1513	.5788	.0144	.0571
.2084	1.0166	.0099	.0856
.2512	1.2523	.0507	.0428
.2726	1.3207	.0839	.0214
.2833	1.3417	.1019	.0107
.2887	1.3490	.0943	.0054
.2913	1.3519	.0691	.0027
.2940	1.3542	.0928	.0027
.2967	1.3561	.1283	.0027
.2994	1.3574	.1809	.0027
.3020	1.3583	.2514	.0027
.3047	1.3587	.3201	.0027
.3074	1.3587	.3390	.0027
.3101	1.3581	.2900	.0027
.3127	1.3572	.2157	.0027
.3154	1.3558	.1531	.0027
.3181	1.3540	.1093	.0027
.3208	1.3518	.0800	.0027
.3234	1.3492	.0602	.0027
.3261	1.3462	.0465	.0040
.3301	1.3411	.0524	.0040
.3341	1.3352	.0383	.0060
.3402	1.3250	.0408	.0090
.3492	1.3072	.0389	.0135
.3627	1.2759	.0315	.0203
.3831	1.2226	.0166	.0305
.4135	1.1422	.0175	.0457
.4364	1.0916	.0531	.0229
.4593	1.0553	.1341	.0229
.4707	1.0434	.1462	.0114
.4764	1.0391	.1170	.0057
.4793	1.0374	.0757	.0029
.4821	1.0359	.0903	.0029
.4850	1.0347	.1073	.0029
.4878	1.0338	.1262	.0029

TABLE 13 (Continued)

.4907	1.0331	.1449	.0029
.4936	1.0326	.1604	.0029
.4964	1.0324	.1688	.0029
.4993	1.0325	.1676	.0029
.5000	1.0325	.1572	.0029

TABLE 14. EXAMPLE 2 - TABULATION OF RESPONSE VERSUS TIME (0.5 TO 2.0 SECONDS) FOR FIGURE 8

T	FOT	OELTA	DELTAT
.5000	1.0325	.1012	.0007
.5007	1.0326	.0368	.0010
.5017	1.0327	.0537	.0010
.5026	1.0329	.0518	.0010
.5036	1.0330	.0497	.0015
.5051	1.0333	.0703	.0015
.5066	1.0337	.0651	.0015
.5081	1.0341	.0599	.0015
.5096	1.0346	.0548	.0015
.5111	1.0352	.0501	.0015
.5125	1.0357	.0456	.0022
.5148	1.0367	.0609	.0022
.5170	1.0379	.0530	.0022
.5192	1.0391	.0461	.0033
.5226	1.0412	.0584	.0033
.5259	1.0435	.0478	.0050
.5309	1.0473	.0566	.0050
.5359	1.0517	.0429	.0075
.5434	1.0589	.0459	.0113
.5547	1.0710	.0408	.0169
.5716	1.0907	.0199	.0254
.5970	1.1197	.0434	.0381
.6160	1.1376	.1109	.0190
.6256	1.1446	.1020	.0095
.6351	1.1499	.1568	.0095
.6446	1.1536	.2408	.0095
.6541	1.1554	.3428	.0095
.6636	1.1552	.3921	.0095
.6731	1.1533	.3325	.0095
.6826	1.1494	.2308	.0095
.6922	1.1439	.1509	.0095
.7017	1.1367	.0995	.0095
.7112	1.1281	.0671	.0095
.7207	1.1182	.0460	.0143
.7350	1.1015	.0430	.0214
.7564	1.0737	.0265	.0321
.7885	1.0314	.0215	.0482
.8126	1.0036	.0697	.0241
.8367	.9821	.1486	.0241
.8487	.9743	.1259	.0120
.8607	.9685	.1791	.0120
.8728	.9648	.2430	.0120
.8848	.9630	.2921	.0120
.8969	.9631	.2895	.0120
.9089	.9648	.2358	.0120
.9210	.9680	.1682	.0120
.9330	.9723	.1112	.0120
.9450	.9775	.0685	.0120
.9571	.9832	.0363	.0181
.9751	.9922	.0068	.0271
1.0022	1.0049	.0850	.0271
1.0158	1.0101	.0883	.0135
1.0293	1.0143	.1226	.0135
1.0361	1.0159	.0743	.0068
1.0429	1.0172	.0822	.0068
1.0497	1.0182	.0888	.0068
1.0564	1.0189	.0933	.0068

TABLE 14 (Continued)

1.0632	1.0192	.0951	.0068
1.0700	1.0193	.0941	.0068
1.0768	1.0190	.0903	.0068
1.0835	1.0184	.0842	.0068
1.0903	1.0175	.0765	.0068
1.0971	1.0164	.0678	.0068
1.1038	1.0150	.0589	.0068
1.1106	1.0134	.0501	.0068
1.1174	1.0116	.0416	.0102
1.1276	1.0085	.0476	.0152
1.1428	1.0035	.0415	.0229
1.1657	.9956	.0036	.0343
1.1999	.9849	.1142	.0343
1.2171	.9809	.1089	.0171
1.2342	.9781	.1382	.0171
1.2428	.9773	.0766	.0086
1.2514	.9767	.0789	.0086
1.2600	.9765	.0788	.0086
1.2685	.9767	.0762	.0086
1.2771	.9771	.0715	.0086
1.2857	.9779	.0651	.0086
1.2943	.9789	.0577	.0086
1.3028	.9802	.0495	.0129
1.3157	.9825	.0586	.0129
1.3285	.9851	.0402	.0193
1.3478	.9896	.0277	.0289
1.3768	.9963	.0243	.0434
1.3985	1.0007	.0636	.0217
1.4202	1.0041	.0947	.0217
1.4419	1.0063	.1143	.0217
1.4636	1.0072	.1178	.0217
1.4853	1.0069	.1046	.0217
1.5070	1.0056	.0792	.0217
1.5287	1.0036	.0478	.0326
1.5612	.9999	.0122	.0488
1.6100	.9954	.0906	.0488
1.6589	.9941	.1487	.0488
1.7077	.9961	.1102	.0488
1.7565	.9998	.0200	.0732
1.8298	1.0037	.1077	.0732
1.9030	1.0028	.1169	.0732
1.9762	.9997	.0022	.1099
2.0000	.9989	.1099	.1099

TABLE 15. EXAMPLE 2 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 9,

$G(s) = 1/(s + 1)^2$

ORDER	COEF
TERMS IN NUMERATOR	
2	1.3400000E-03
1	2.0670000E-01
0	1.0000000E 00
TERMS IN DENOMINATOR	
6	6.9981000E-07
5	1.1622881E-04
4	1.7955290E-03
3	3.9960000E-02
2	2.9758000E-01
1	1.2593000E 00
0	1.0000000E 00

ALPHA	BETA	K	INVERSE LAPLACE FORM
1.0000000E 00	0.0000000E 00	1	
1.0000000E 00	0.0000000E 00	1	(1.0221570E 00)*EXP(-ALPHA*T)
0.0000000E 00			
4.1257829E 00	4.2935266E 00	1	
4.1257829E 00	4.2935266E 00	1	(1.4910578E 00)*EXP(-ALPHA*T)*SIN((BETA*T)+(-7.7775463E-01))
2.6566636E 00	1.6091091E 01	1	
2.6566636E 00	1.6091091E 01	1	(-4.8490544E-01)*EXP(-ALPHA*T)*SIN((BETA*T)+(-4.9719073E-02))
1.5152134E 02	0.0000000E 00	1	
1.5152134E 02	0.0000000E 00	1	(-8.6702246E-06)*EXP(-ALPHA*T)
0.0000000E 00			
0.0000000E 00	0.0000000E 00	0	

TABLE 16. EXAMPLE 2 - TABULATION OF RESPONSE VERSUS TIME (0 TO 3.0 SECONDS) FOR FIGURE 9

T	FOT	DELTA	DELTAT
.0000	.0000	.0552	.0007
.0007	.0000	.0008	.0010
.0016	.0000	.0043	.0015
.0031	.0000	.0134	.0022
.0054	.0000	.0353	.0033
.0087	.0002	.0854	.0033
.0120	.0005	.1196	.0033
.0154	.0011	.1440	.0033
.0187	.0020	.1539	.0033
.0221	.0032	.1488	.0033
.0254	.0048	.1330	.0033
.0288	.0069	.1128	.0033
.0321	.0095	.0927	.0033
.0354	.0127	.0752	.0033
.0388	.0164	.0608	.0033
.0421	.0208	.0494	.0050
.0471	.0286	.0577	.0050
.0521	.0380	.0434	.0075
.0597	.0553	.0470	.0113
.0709	.0886	.0455	.0169
.0878	.1557	.0386	.0254
.1132	.2923	.0277	.0381
.1513	.5537	.0133	.0571
.2084	.9483	.0169	.0856
.2512	1.1388	.0873	.0428
.2726	1.1822	.2266	.0214
.2940	1.1902	1.8545	.0214
.3154	1.1664	.5124	.0214
.3368	1.1170	.0987	.0214
.3582	1.0497	.0323	.0321
.3903	.9338	.0084	.0482
.4385	.7774	.0502	.0482
.4526	.7233	.0792	.0241
.4867	.6891	.1940	.0241
.5107	.6737	.5789	.0241
.5348	.6733	.9407	.0241
.5589	.6821	.3487	.0241
.5830	.6939	.0045	.0361
.6011	.7012	.1926	.0181
.6191	.7046	.4473	.0181
.6372	.7028	.6080	.0181
.6552	.6947	.4056	.0181
.6733	.6800	.2029	.0181
.6914	.6590	.1047	.0181
.7094	.6327	.0571	.0181
.7275	.6022	.0308	.0271
.7546	.5521	.0157	.0406
.7952	.4781	.0297	.0610
.8257	.4321	.0755	.0305
.8409	.4138	.0665	.0152
.8562	.3990	.0974	.0152
.8714	.3876	.1419	.0152
.8867	.3796	.2004	.0152
.9019	.3746	.2563	.0152
.9171	.3722	.2704	.0152
.9324	.3718	.2220	.0152
.9476	.3727	.1403	.0152

TABLE 16 (Continued)

.9629	.3745	.0588	.0152
.9781	.3764	.0133	.0229
1.0010	.3784	.1388	.0229
1.0238	.3782	.2485	.0229
1.0467	.3747	.2785	.0229
1.0695	.3676	.2253	.0229
1.0924	.3572	.1482	.0229
1.1153	.3441	.0843	.0229
1.1381	.3293	.0365	.0343
1.1724	.3066	.0163	.0514
1.1981	.2910	.0652	.0257
1.2239	.2779	.1141	.0257
1.2367	.2726	.0759	.0129
1.2496	.2681	.0869	.0129
1.2624	.2644	.0947	.0129
1.2753	.2614	.0974	.0129
1.2881	.2593	.0939	.0129
1.3010	.2577	.0840	.0129
1.3139	.2566	.0690	.0129
1.3267	.2560	.0505	.0129
1.3396	.2556	.0306	.0193
1.3589	.2552	.0088	.0289
1.3878	.2543	.0603	.0289
1.4167	.2520	.1203	.0289
1.4457	.2477	.1379	.0289
1.4746	.2413	.1176	.0289
1.5035	.2330	.0775	.0289
1.5325	.2237	.0332	.0434
1.5759	.2095	.0265	.0651
1.6084	.2000	.0690	.0325
1.6410	.1922	.0939	.0325
1.6735	.1863	.0951	.0325
1.7061	.1820	.0713	.0325
1.7386	.1785	.0318	.0488
1.7874	.1734	.0253	.0732
1.8607	.1630	.1066	.0732
1.9339	.1492	.0415	.1099
2.0438	.1306	.1105	.1099
2.1536	.1189	.0743	.1099
2.2635	.1075	.0387	.1648
2.4282	.0894	.0447	.2472
2.6754	.0707	.0324	.3708
3.0000	.0510	.0496	.5561

TABLE 17. EXAMPLE 2 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 10, $G(s) = 1/(s + 1)^2$

ORDER	COEFF
TERMS IN NUMERATOR	
2	1.3400000E-03
1	2.0670000E-01
0	1.0000000E 00
TERMS IN DENOMINATOR	
7	6.9981000E-07
6	1.1692862E-04
5	1.9117578E-03
4	4.1755529E-02
3	3.3754000E-01
2	1.5568800E 00
1	2.2593000E 00
0	1.0000000E 00

ALPHA	BETA	K	INVERSE LAPLACE FORM
1.0000000E 00	0.0000000E 00	2	
1.0000000E 00	0.0000000E 00	2	(1.6140854E-02)*EXP(-ALPHA*T)+(1.0221570E 00)*T*EXP(-ALPHA*T)
4.1257829E 00	4.2935266E 00	1	
4.1257829E 00	4.2935266E 00	1	(-2.8075808E-01)*EXP(-ALPHA*T)*SIN((BETA*T)+(1.6375535E-01))
2.6566636E 00	1.6091091E 01	1	
2.6566636E 00	1.6091091E 01	1	(2.9976572E-02)*EXP(-ALPHA*T)*SIN((BETA*T)+(1.4184833E 00))
1.5152134E 02	0.0000000E 00	1	
1.5152134E 02	0.0000000E 00	1	(5.7601300E-08)*EXP(-ALPHA*T)
0.0000000E 00			
0.0000000E 00	0.0000000E 00	0	

TABLE 18. EXAMPLE 2 - TABULATION OF RESPONSE VERSUS TIME (0 TO 3.0 SECONDS) FOR FIGURE 10

T	FOT	DELTA	DELTAT
.0000	.0000	.0652	.0007
.0007	.0000	.0000	.0010
.0016	.0000	.0000	.0015
.0031	.0000	.0000	.0022
.0054	.0000	.0001	.0033
.0087	.0000	.0003	.0050
.0137	.0000	.0012	.0075
.0212	.0000	.0041	.0113
.0325	.0001	.0140	.0169
.0494	.0004	.0449	.0254
.0748	.0020	.1331	.0254
.0875	.0036	.0947	.0127
.1002	.0059	.1073	.0127
.1129	.0090	.1125	.0127
.1255	.0130	.1098	.0127
.1382	.0181	.1010	.0127
.1509	.0242	.0885	.0127
.1636	.0315	.0750	.0127
.1763	.0398	.0619	.0127
.1890	.0491	.0501	.0127
.2017	.0594	.0398	.0190
.2207	.0765	.0436	.0285
.2492	.1047	.0365	.0428
.2920	.1495	.0049	.0642
.3563	.2111	.0866	.0642
.3884	.2360	.0827	.0321
.4205	.2565	.1002	.0321
.4526	.2731	.0990	.0321
.4847	.2871	.0750	.0321
.5168	.2995	.0388	.0482
.5650	.3172	.0066	.0722
.6372	.3438	.0116	.1084
.6914	.3618	.0864	.0542
.7185	.3692	.0749	.0271
.7456	.3752	.0916	.0271
.7727	.3797	.0997	.0271
.7998	.3828	.0963	.0271
.8269	.3846	.0820	.0271
.8540	.3854	.0607	.0271
.8811	.3855	.0377	.0406
.9217	.3851	.0198	.0610
.9827	.3844	.0114	.0914
1.0741	.3836	.0207	.1372
1.2113	.3761	.1466	.1372
1.3484	.3617	.0271	.2057
1.5542	.3394	.0317	.3086
1.8628	.2980	.0232	.4629
2.3257	.2339	.0030	.6944
3.0000	.1535	.0612	.6944

TABLE 19. EXAMPLE 2 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 11,

$G(s) = 1/(0.111s^2 + 0.266s + 1)$

ORDER	COEF
TERMS IN NUMERATOR	
2	1.3400000E-03
1	2.0670000E-01
0	1.0000000E 00
TERMS IN DENOMINATOR	
7	7.7748891E-08
6	1.3021883E-05
5	2.1815477E-04
4	4.8154258E-03
3	4.0695975E-02
2	2.1852494E-01
1	5.2596000E-01
0	1.0000000E 00

ALPHA	BETA	K	INVERSE LAPLACE FORM
1.2000900E 00	2.7496698E 00	1	
1.2000900E 00	2.7496698E 00	1	(4.5255096E 00)*EXP(-ALPHA*T)*SIN((BETA*T)+(-9.6448637E-02))
4.1257829E 00	4.2935266E 00	1	
4.1257829E 00	4.2935266E 00	1	(-2.8251095E 00)*EXP(-ALPHA*T)*SIN((BETA*T)+(-5.6620545E-02))
2.6566636E 00	1.6091091E 01	1	
2.6566636E 00	1.6091091E 01	1	(2.7838807E-01)*EXP(-ALPHA*T)*SIN((BETA*T)+(1.4377687E 00))
1.5152134E 02	0.0000000E 00	1	
1.5152134E 02	0.0000000E 00	1	(5.1967076E-07)*EXP(-ALPHA*T)
0.0000000E 00			
0.0000000E 00	0.0000000E 00	0	

TABLE 20. EXAMPLE 2 - TABULATION OF RESPONSE VERSUS TIME (0 TO 3.0 SECONDS) FOR FIGURE 11

T	FOT	DELTA	DELTAT
.0000	.0000	.0584	.0007
.0007	.0000	.0000	.0010
.0016	.0000	.0000	.0015
.0031	.0000	.0001	.0022
.0054	.0000	.0007	.0033
.0087	.0000	.0028	.0050
.0137	.0000	.0105	.0075
.0212	.0001	.0370	.0113
.0325	.0007	.1233	.0113
.0438	.0023	.2162	.0113
.0551	.0056	.2722	.0113
.0663	.0114	.2511	.0113
.0776	.0205	.1863	.0113
.0889	.0339	.1258	.0113
.1002	.0522	.0837	.0113
.1114	.0763	.0567	.0113
.1227	.1068	.0394	.0169
.1396	.1656	.0391	.0254
.1650	.2845	.0337	.0381
.2031	.5286	.0239	.0571
.2601	.9932	.0099	.0856
.3458	1.6554	.0222	.1284
.4100	1.9495	.0863	.0642
.4421	2.0220	.1377	.0321
.4582	2.0429	.1481	.0161
.4742	2.0555	.2777	.0161
.4903	2.0613	.4951	.0161
.5063	2.0617	.5700	.0161
.5224	2.0580	.3667	.0161
.5384	2.0513	.1936	.0161
.5545	2.0422	.1092	.0161
.5706	2.0314	.0712	.0161
.5866	2.0190	.0555	.0161
.6027	2.0052	.0510	.0161
.6187	1.9895	.0509	.0161
.6348	1.9716	.0512	.0161
.6508	1.9510	.0495	.0241
.6749	1.9137	.0662	.0241
.6990	1.8670	.0531	.0241
.7231	1.8096	.0399	.0361
.7592	1.7016	.0400	.0542
.8134	1.4943	.0308	.0813
.8947	1.1245	.0107	.1219
1.0166	.5953	.0207	.1829
1.1995	-.0349	.0345	.2743
1.3366	-.3854	.0919	.1372
1.4052	-.4942	.1244	.0686
1.4395	-.5323	.0978	.0343
1.4738	-.5612	.1297	.0343
1.4909	-.5726	.0791	.0171
1.5081	-.5822	.0901	.0171
1.5252	-.5902	.1026	.0171
1.5424	-.5965	.1164	.0171
1.5595	-.6013	.1314	.0171
1.5767	-.6047	.1462	.0171
1.5938	-.6066	.1586	.0171
1.6110	-.6071	.1657	.0171

TABLE 20 (Continued)

1.6281	-.6062	.1649	.0171
1.6452	-.6038	.1559	.0171
1.6624	-.6000	.1405	.0171
1.6795	-.5948	.1219	.0171
1.6967	-.5881	.1030	.0171
1.7138	-.5800	.0855	.0171
1.7310	-.5706	.0702	.0171
1.7481	-.5598	.0573	.0171
1.7653	-.5479	.0466	.0257
1.7910	-.5279	.0538	.0257
1.8167	-.5057	.0389	.0386
1.8553	-.4692	.0382	.0579
1.9131	-.4096	.0282	.0868
1.9999	-.3160	.0096	.1302
2.1301	-.1762	.0150	.1953
2.3254	.0047	.1349	.1953
2.4231	.0693	.1458	.0976
2.4719	.0944	.0931	.0488
2.5207	.1149	.1077	.0488
2.5695	.1310	.1241	.0488
2.6184	.1428	.1403	.0488
2.6428	.1472	.0750	.0244
2.6672	.1505	.0767	.0244
2.6916	.1528	.0769	.0244
2.7160	.1542	.0755	.0244
2.7404	.1547	.0724	.0244
2.7648	.1543	.0681	.0244
2.7893	.1531	.0628	.0244
2.8137	.1512	.0569	.0244
2.8381	.1485	.0510	.0244
2.8625	.1453	.0452	.0366
2.8991	.1394	.0578	.0366
2.9357	.1324	.0472	.0549
2.9907	.1203	.0541	.0549
3.0000	.1181	.0375	.0824

TABLE 21. EXAMPLE 3 - INPUT AND ANALYTIC TIME SOLUTION FOR FIGURE 12.

$G(s) = 1/s^2$

ORDER	COEF
TERMS IN NUMERATOR	
4	2.4666500E 01
3	6.0177660E 02
2	7.1376580E 05
1	1.3620729E 07
0	4.7818957E 08
TERMS IN DENOMINATOR	
6	1.0000000E 00
5	9.2000000E 00
4	2.6019200E 04
3	1.5488000E 05
2	1.0240000E 07
1	0.0000000E 00
0	0.0000000E 00

ALPHA	BETA	K	INVERSE LAPLACE FORM
0.0000000E 00	0.0000000E 00	2	(6.2383904E-01)+(4.6698200E 01)*T
3.0000000E 00	1.9773720E 01	1	
3.0000000E 00	1.9773720E 01	1	(-1.2511746E 00)*EXP(-ALPHA*T)*SIN((BETA*T)+(5.2198604E-01))
1.6000000E 00	1.5999200E 02	1	
1.6000000E 00	1.5999200E 02	1	(-1.5360143E-02)*EXP(-ALPHA*T)*SIN((BETA*T)+(1.3923601E-08))
0.0000000E 00	0.0000000E 00	0	

TABLE 22. EXAMPLE 3 - TABULATION OF RESPONSE VERSUS TIME (0 TO 0.1 SECOND) FOR FIGURE 12

T	FOT	DELTA	DELTAT
.0000	.0000	.0064	.0006
.0006	.0155	.0002	.0009
.0016	.0390	.0003	.0014
.0030	.0752	.0006	.0021
.0051	.1315	.0009	.0032
.0082	.2218	.0014	.0047
.0130	.3720	.0019	.0071
.0201	.6271	.0017	.0107
.0308	1.0437	.0006	.0160
.0468	1.7173	.0019	.0240
.0708	2.9950	.0018	.0360
.1000	4.7422	.0012	.0541

TABLE 23. EXAMPLE 3 - TABULATION OF RESPONSE VERSUS TIME (0 TO 0.01 SECOND) FOR FIGURE 13

T	FOT	DELTA	DELTAT
.0000	.0000	.0036	.0006
.0006	.0155	.0002	.0009
.0016	.0390	.0003	.0014
.0030	.0752	.0006	.0021
.0051	.1315	.0009	.0032
.0082	.2218	.0014	.0047
.0100	.2753	.0019	.0071

TABLE 24. EXAMPLE 3 - TABULATION OF RESPONSE VERSUS TIME (0 TO 10.0 SECONDS) FOR FIGURE 14

T	FOT	DELTA	DELTAT
.0000	.0000	.0000	.0006
.0006	.0155	.0002	.0009
.0016	.0390	.0003	.0014
.0030	.0752	.0006	.0021
.0051	.1315	.0009	.0032
.0082	.2218	.0014	.0047
.0130	.3720	.0019	.0071
.0201	.6271	.0017	.0107
.0308	1.0437	.0006	.0160
.0468	1.7173	.0019	.0240
.0708	2.9950	.0018	.0360
.1069	5.1873	.0012	.0541
.1609	8.5445	.0010	.0811
.2420	12.4187	.0045	.1216
.3637	17.1824	.0026	.1825
.5461	26.3617	.0008	.2737
.8198	39.0033	.0008	.4105
1.2304	58.0875	.0002	.6158
1.8462	86.8414	.0001	.9237
2.7699	129.9752	.0001	1.3856
4.1555	194.6796	.0000	2.0784
6.2339	291.7366	.0000	3.1176
9.3515	437.3222	.0000	4.6764
10.0000	467.6058	.0000	7.0146

START TIME= .0000 ACTUAL STOP TIME= 2.5000 PROPOSED STOP TIME= 2.5000

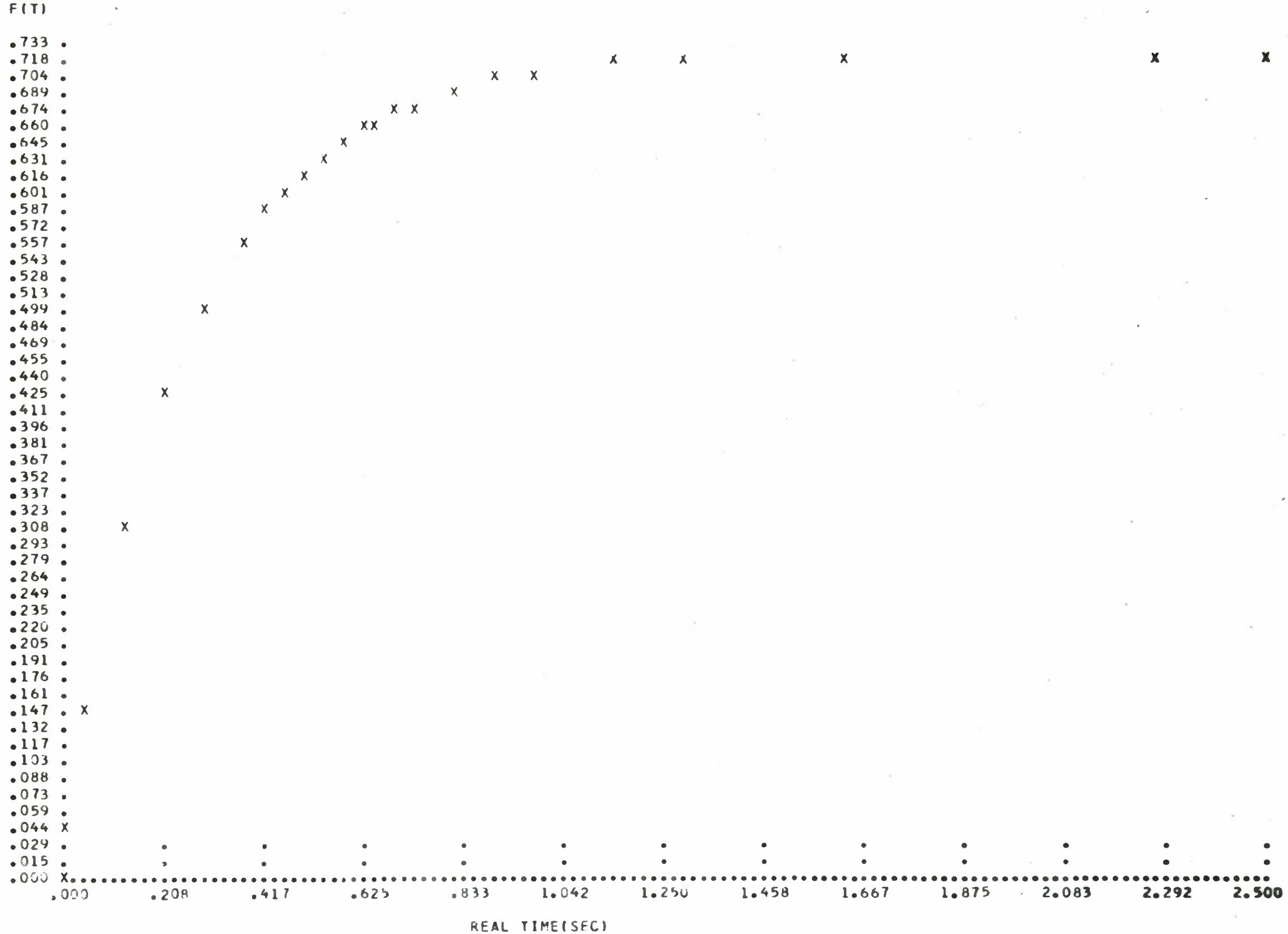

FIGURE 2. TRANSIENT RESPONSE OF EXAMPLE 1 TO A STEP INPUT,
t = 0 TO 2.5 SECONDS

START TIME= .0000 ACTUAL STOP TIME= 5.0000 PROPOSED STOP TIME= 5.0000

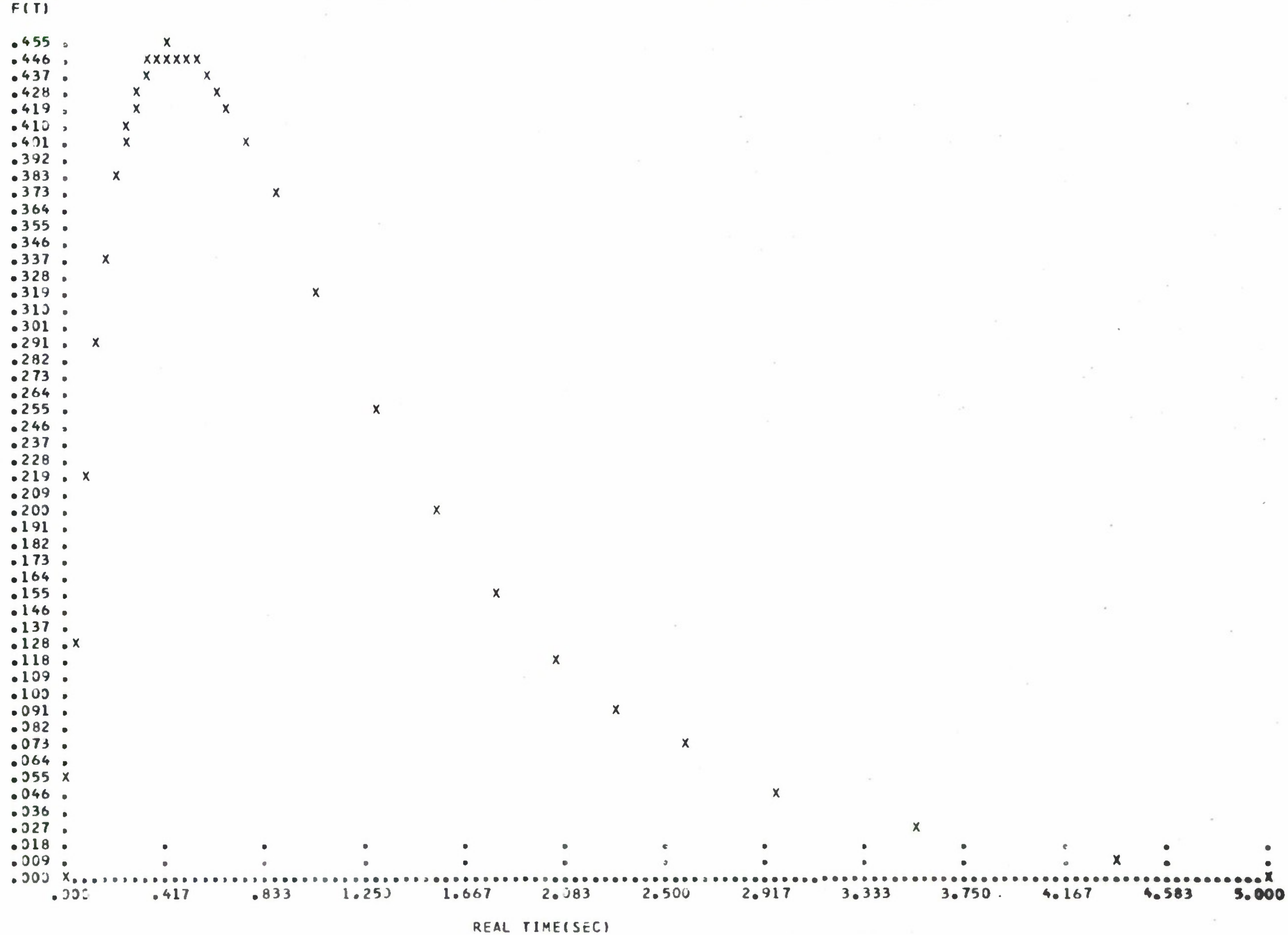

FIGURE 3. TRANSIENT RESPONSE OF EXAMPLE 1 TO AN EXPONENTIAL LAG.
$t = 0$ TO 5.0 SECONDS

START TIME= .0000 ACTUAL STOP TIME= 5.0000 PROPOSED STOP TIME= 5.0000

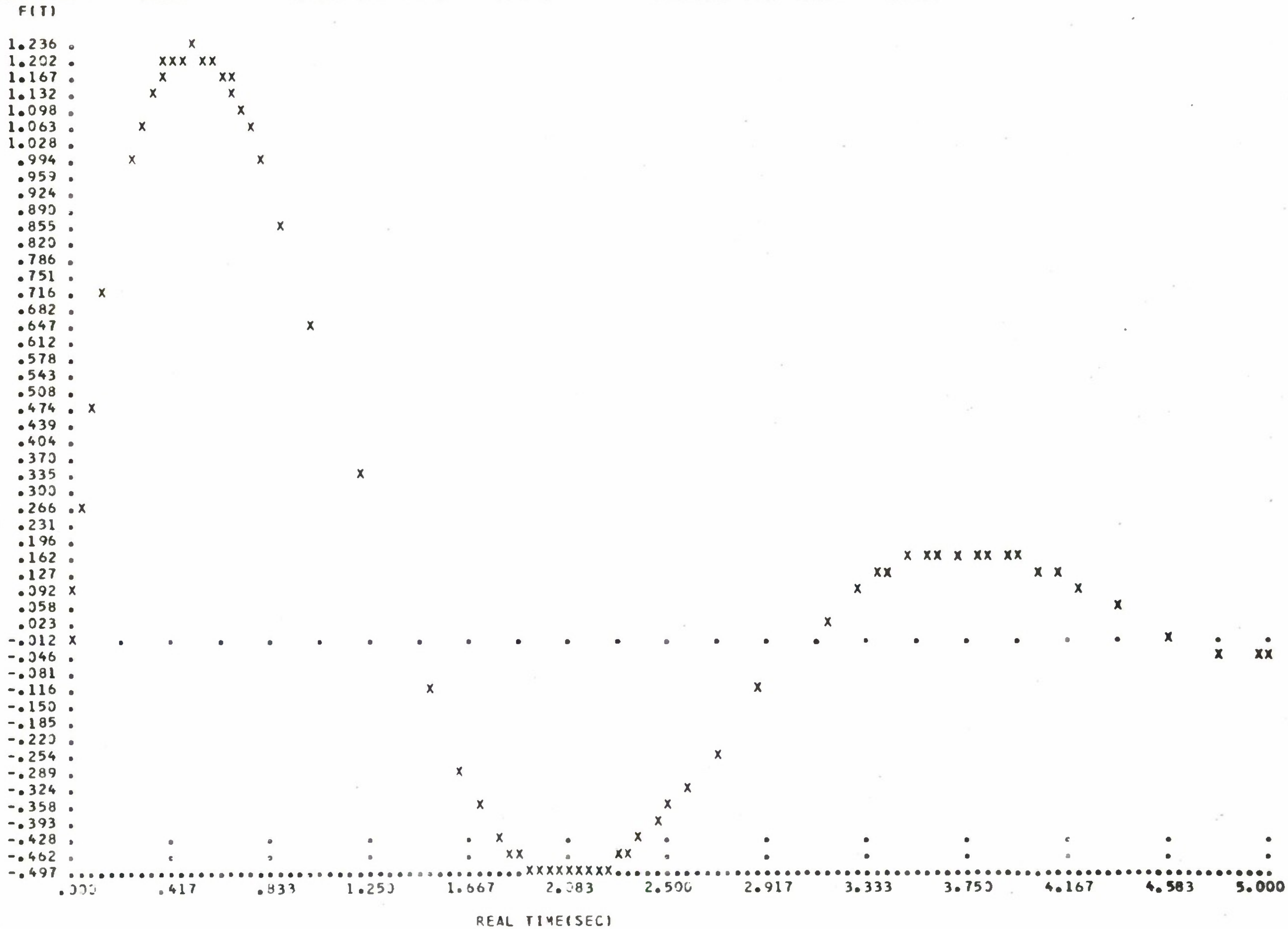

FIGURE 4. TRANSIENT RESPONSE OF EXAMPLE 1 TO A DAMPED SINUSOID.
t = 0 TO 5.0 SECONDS

START TIME= .0000 ACTUAL STOP TIME= 5.0000 PROPOSED STOP TIME= 5.0000

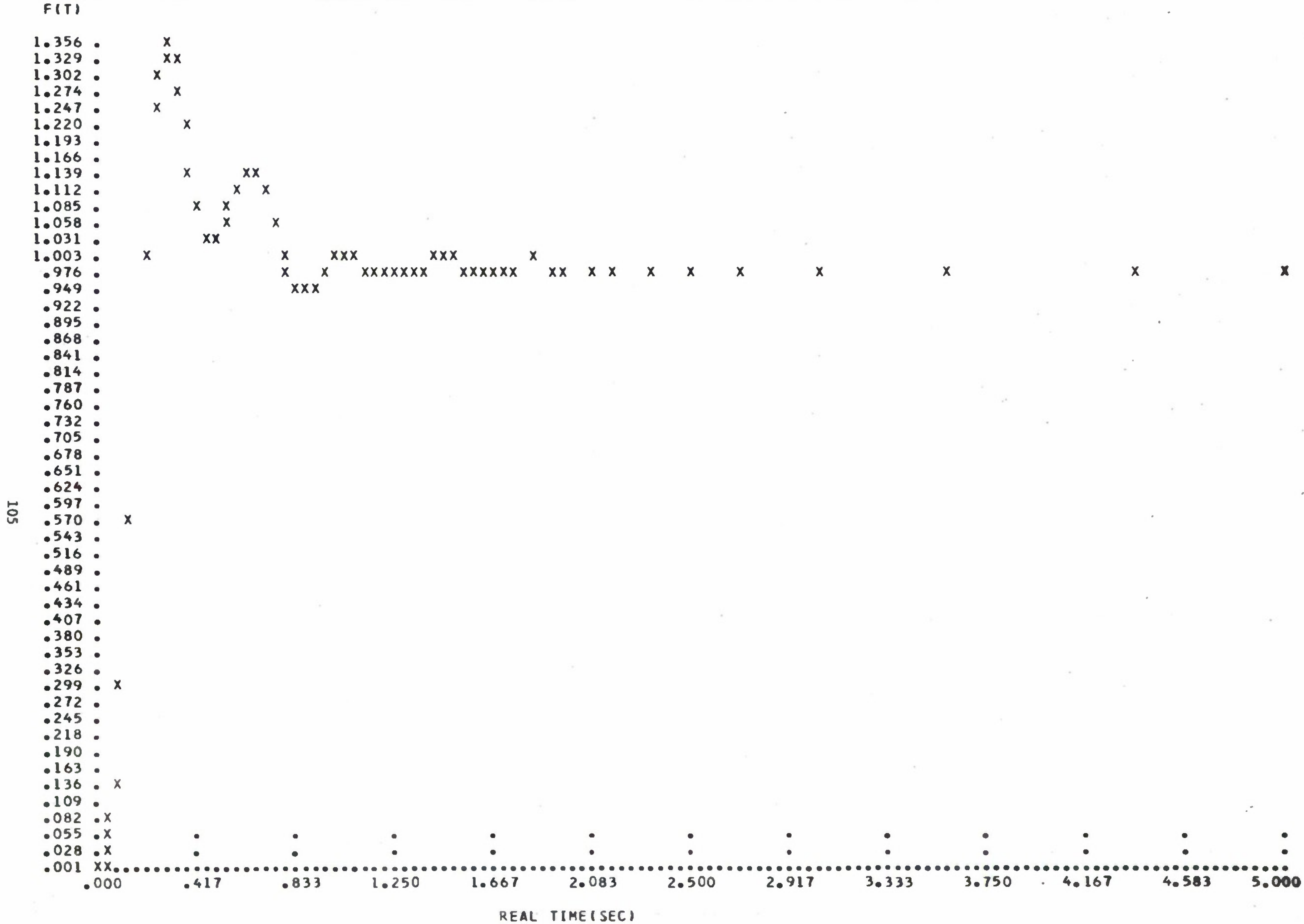

FIGURE 5. TRANSIENT RESPONSE OF EXAMPLE 2 TO A STEP INPUT,
t = 0 TO 5.0 SECONDS

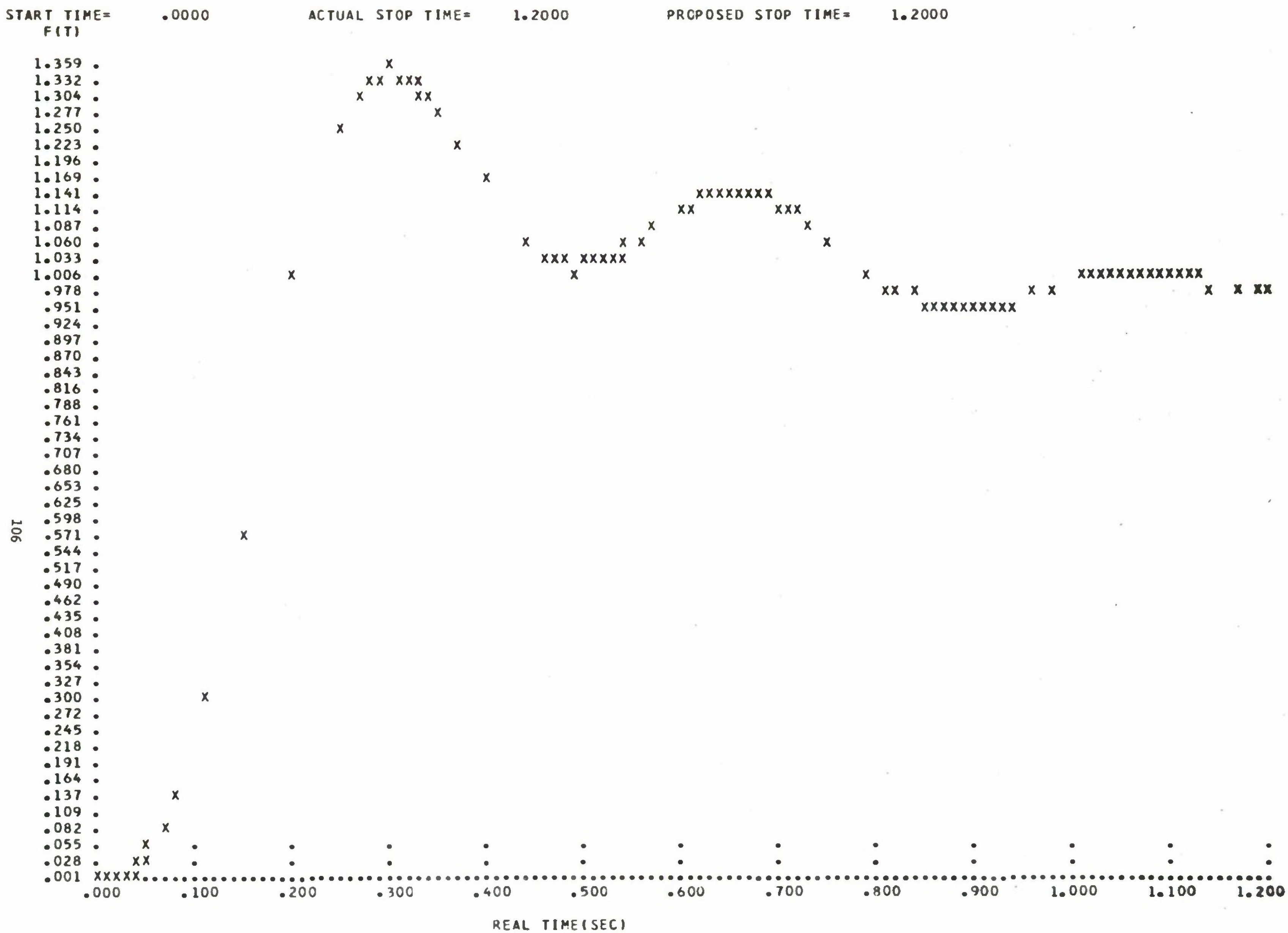

FIGURE 6. TRANSIENT RESPONSE OF EXAMPLE 2 TO A STEP INPUT,
t = 0 TO 1.2 SECONDS

START TIME= .0000 ACTUAL STOP TIME= .5000 PROPOSED STOP TIME= .5000

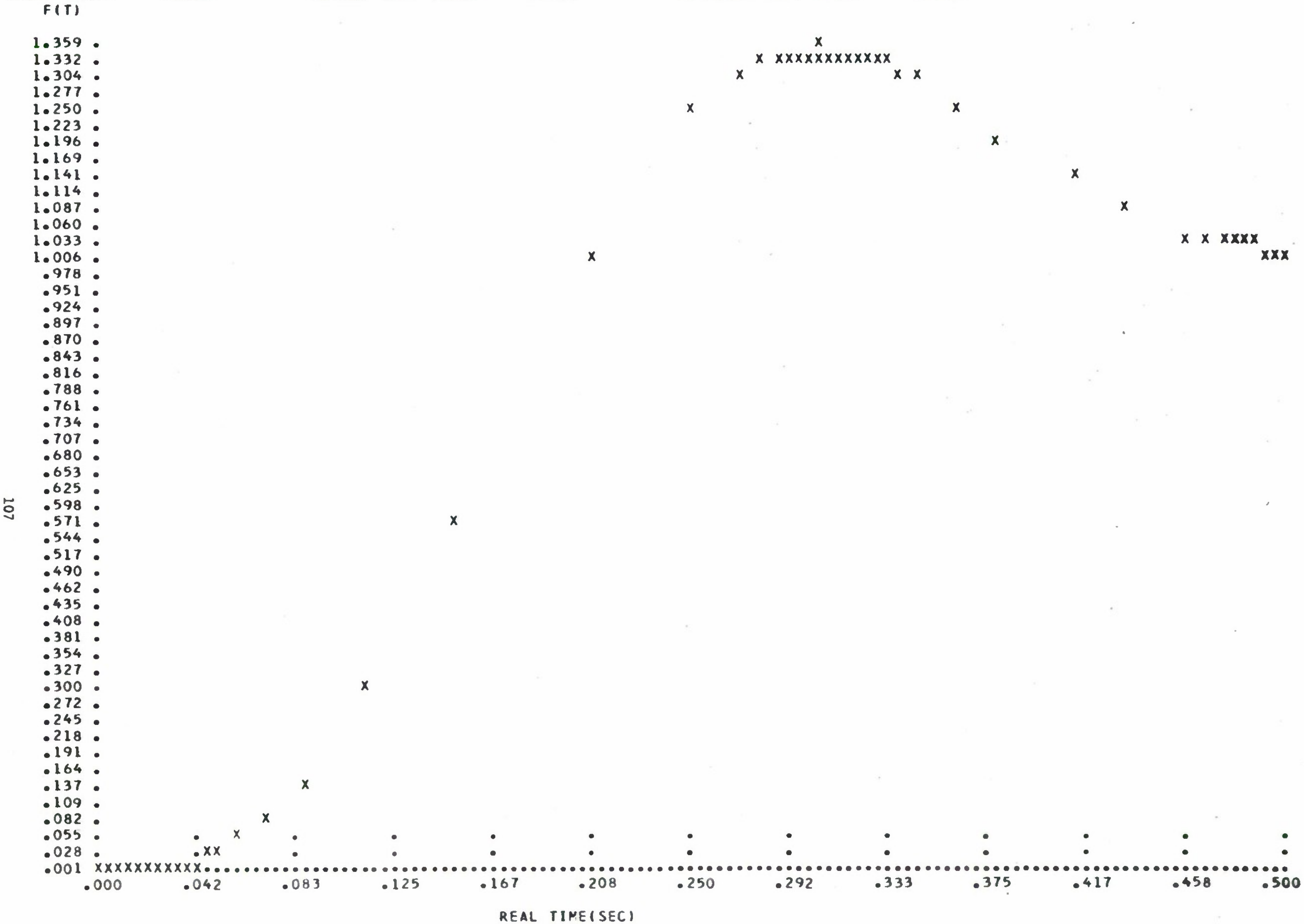

FIGURE 7. TRANSIENT RESPONSE OF EXAMPLE 2 TO A STEP INPUT, t = 0 TO 0.5 SECOND

START TIME= .5000 ACTUAL STOP TIME= 2.0000 PROPOSED STOP TIME= 2.0000

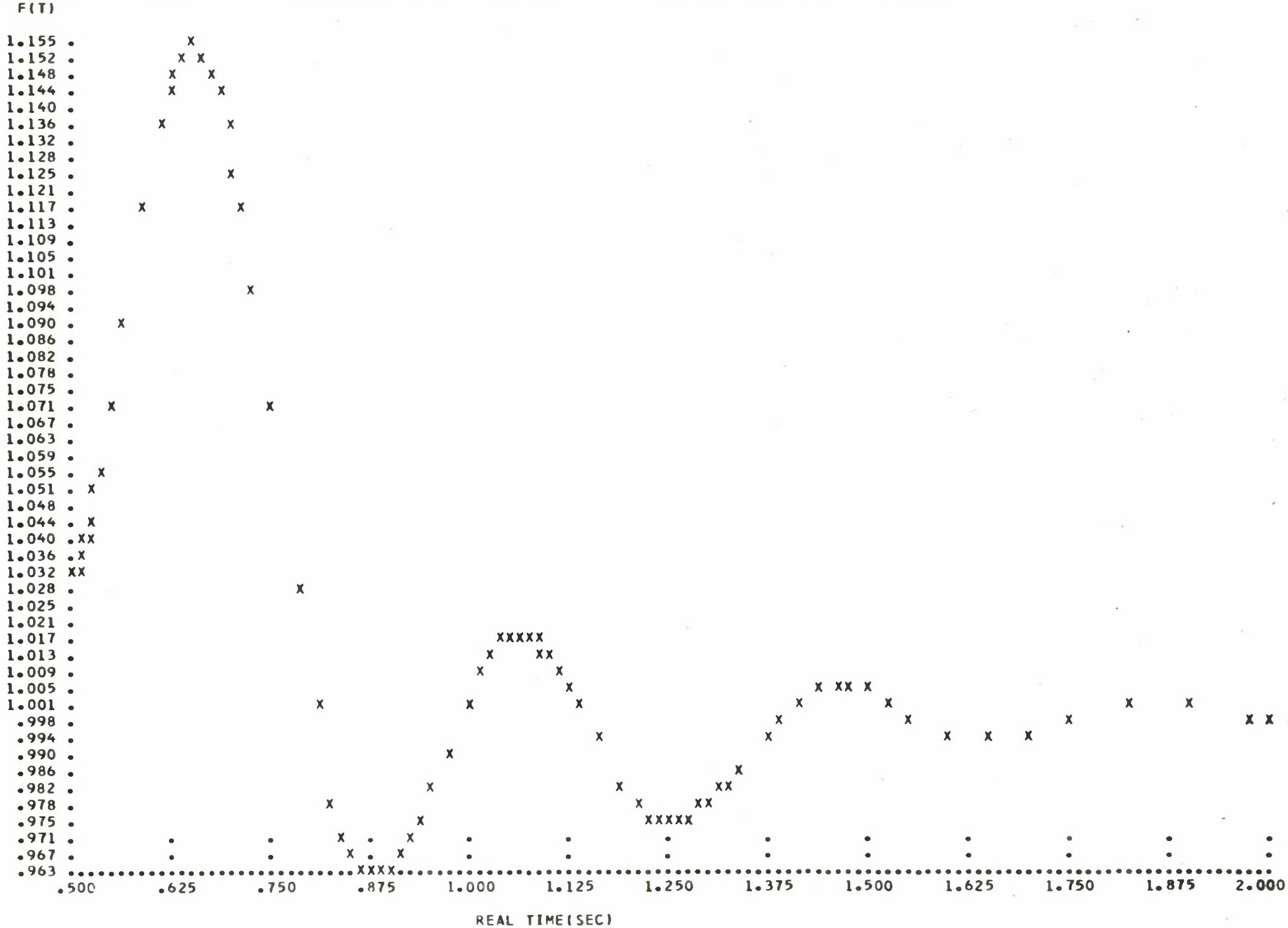

FIGURE 8. TRANSIENT RESPONSE OF EXAMPLE 2 TO A STEP INPUT,
t = 0.5 TO 2.0 SECONDS

START TIME= .0000 ACTUAL STOP TIME= 3.0000 PROPOSED STOP TIME= 3.0000

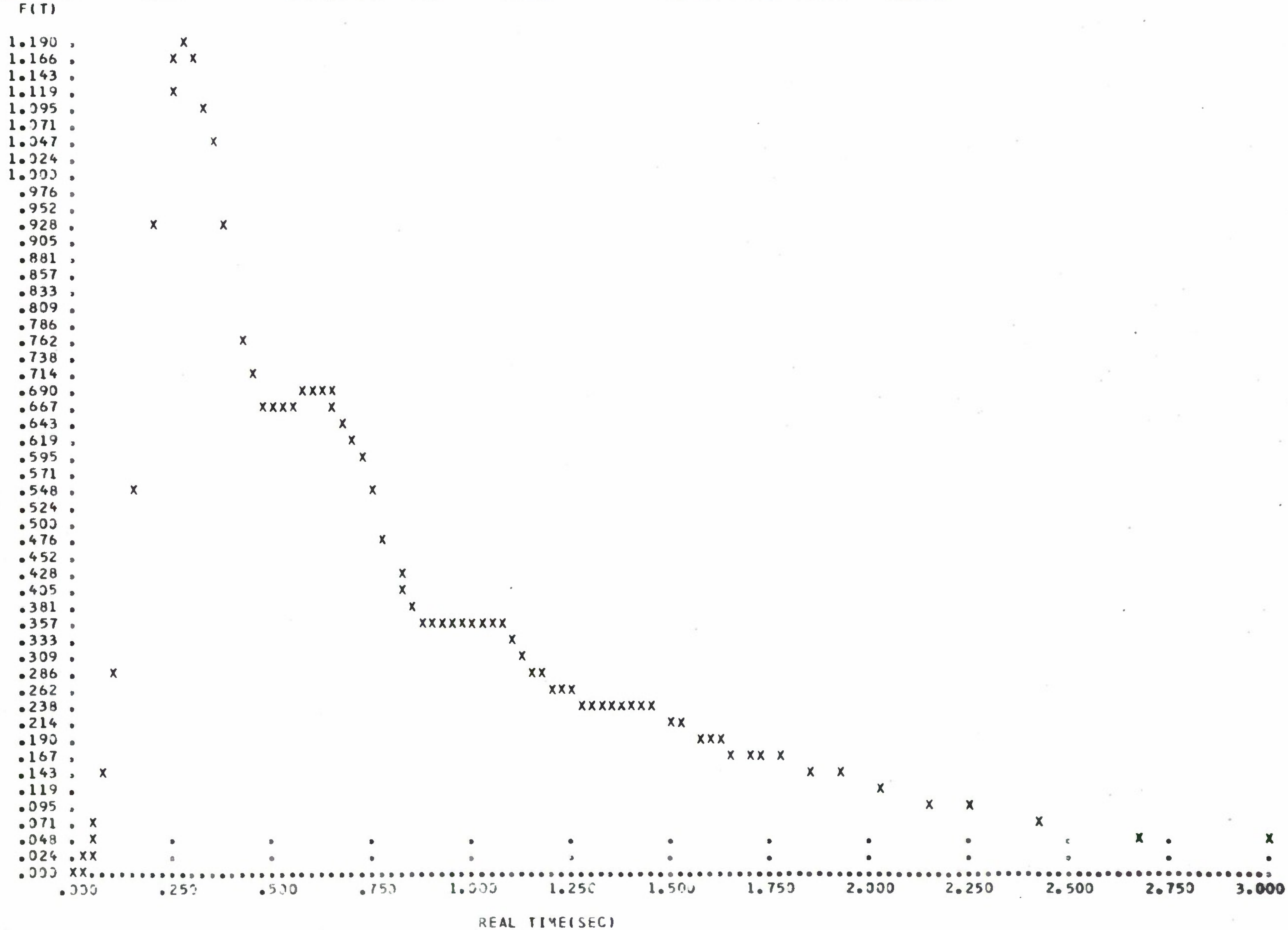

FIGURE 9. TRANSIENT RESPONSE OF EXAMPLE 2 TO AN EXPONENTIAL LAG,
t = 0 TO 3.0 SECONDS

START TIME= .0000 ACTUAL STOP TIME= 3.0000 PROPOSED STOP TIME= 3.0000

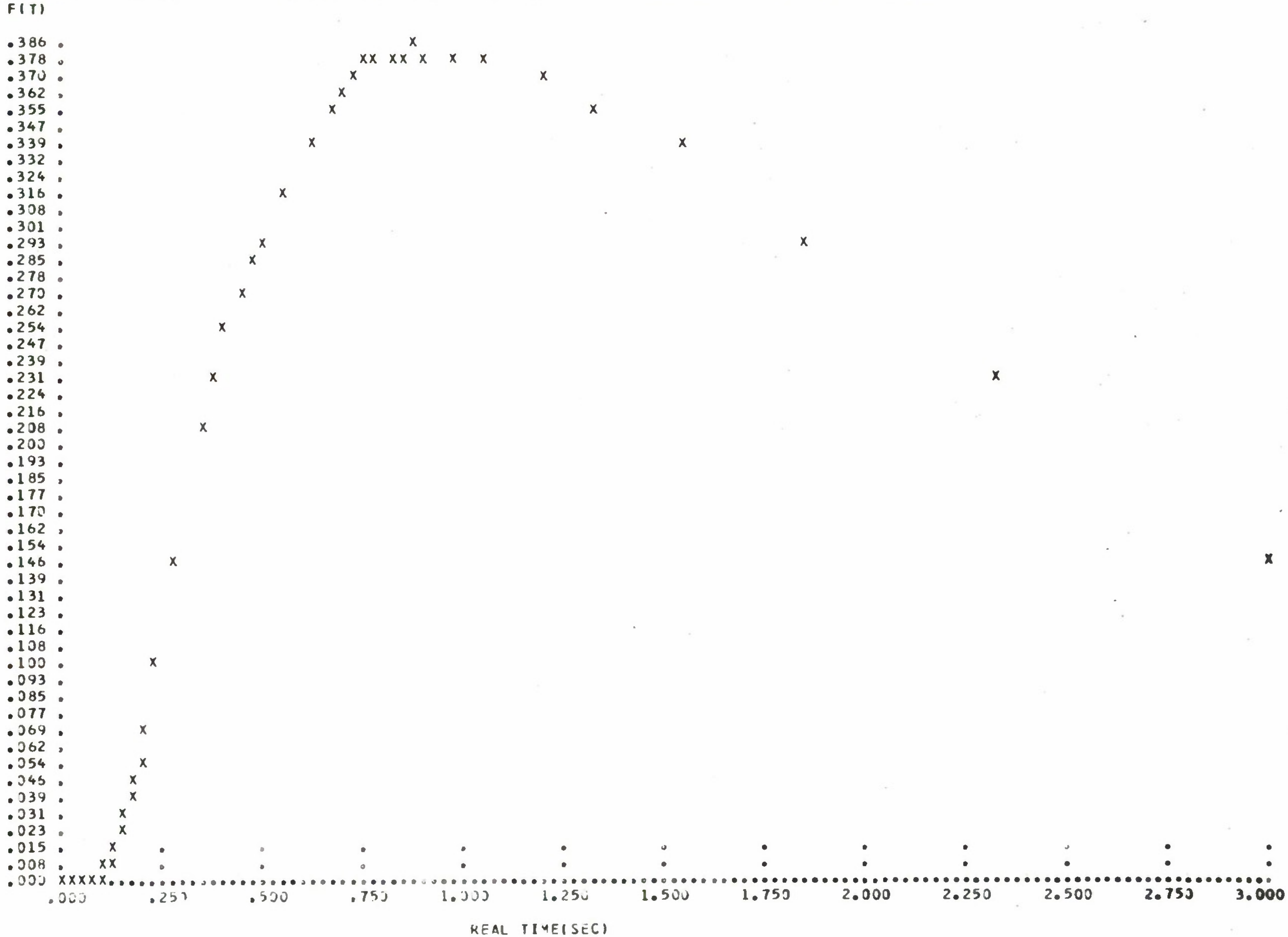

FIGURE 10. TRANSIENT RESPONSE OF EXAMPLE 2 TO A SECOND ORDER EXPONENTIAL LAG, t = 0 TO 3.0 SECONDS

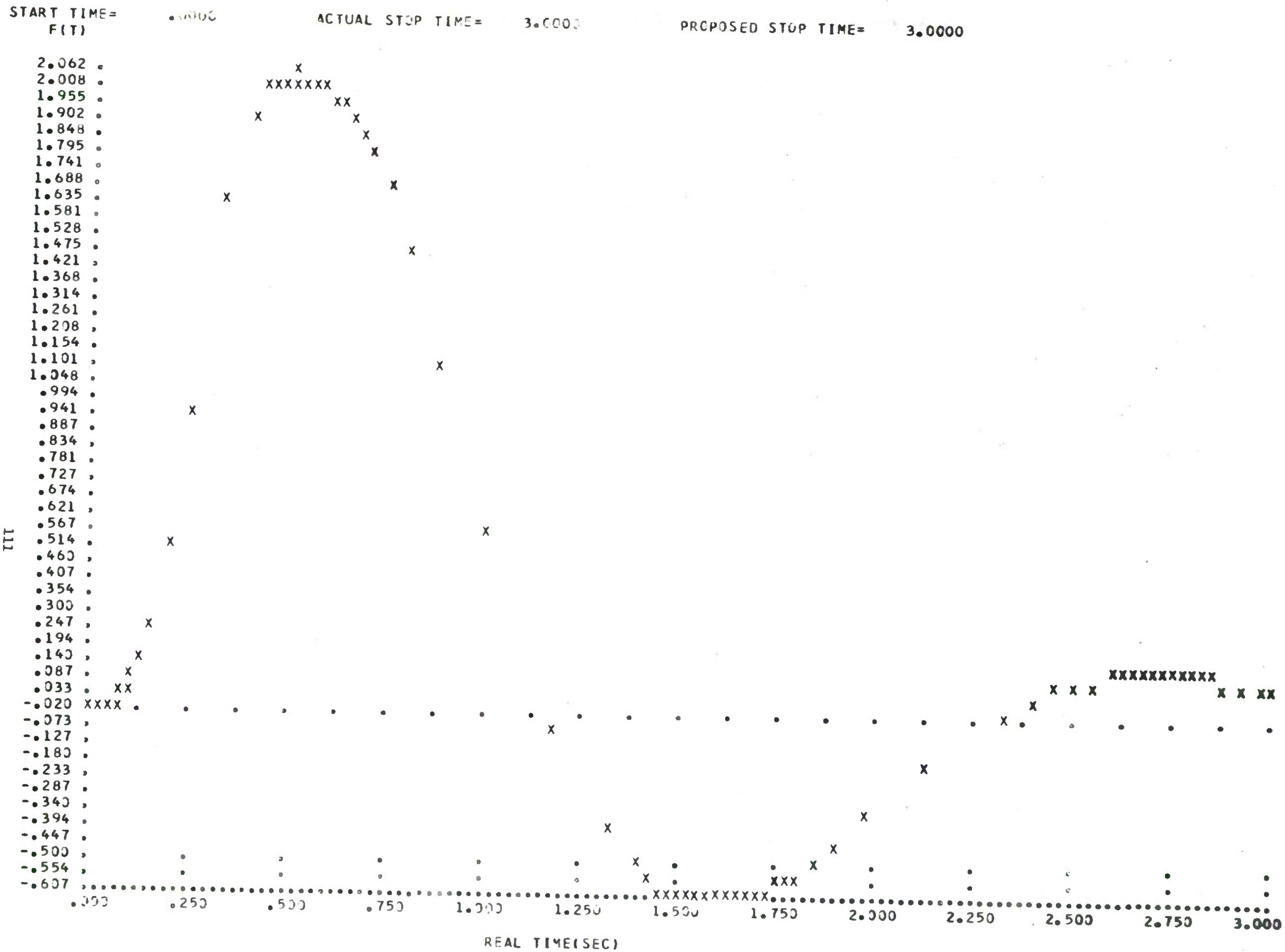

FIGURE 11. TRANSIENT RESPONSE OF EXAMPLE 2 TO A DAMPED SINUSOID,
t = 0 TO 3.0 SECONDS

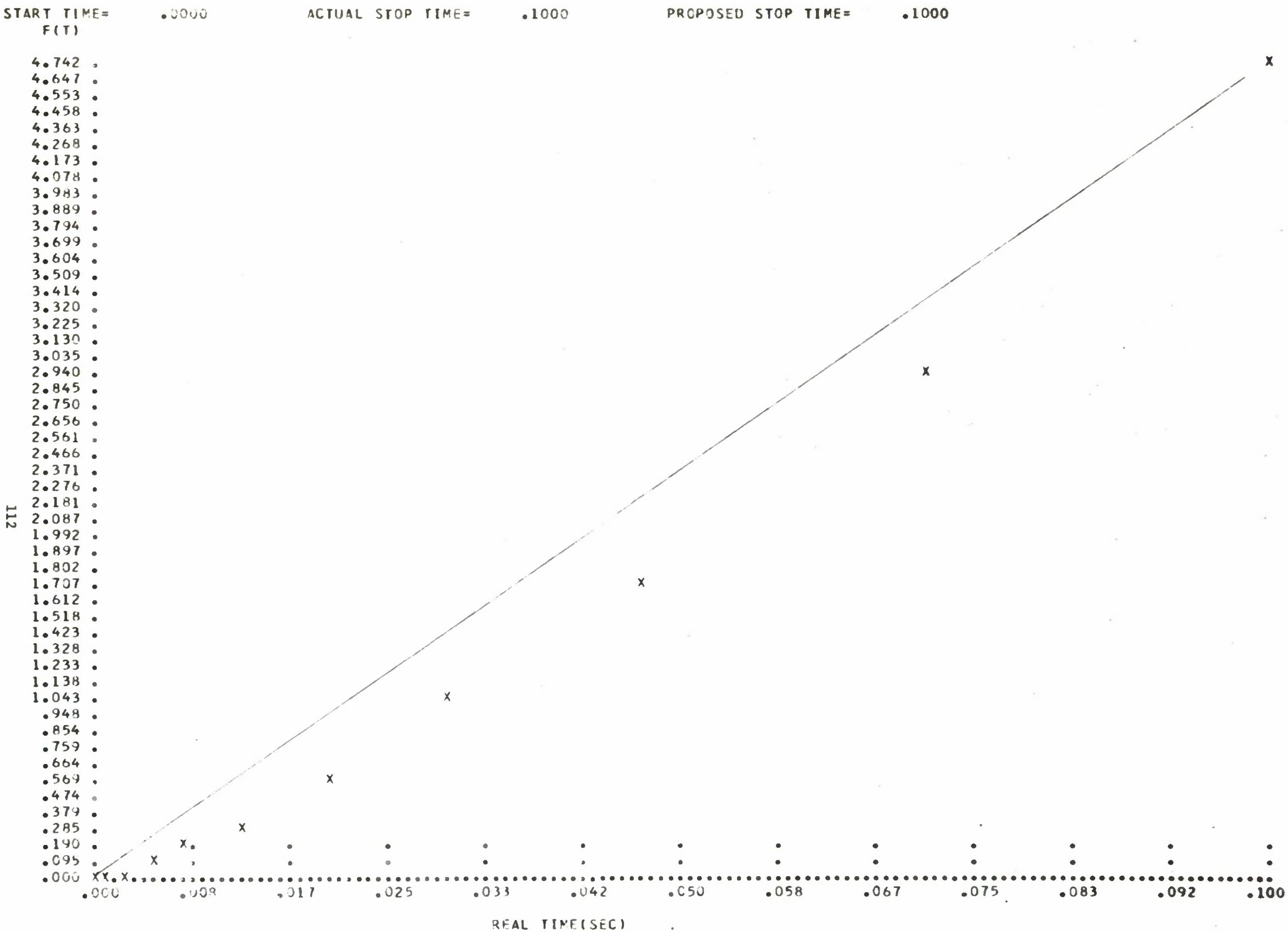

FIGURE 12. TRANSIENT RESPONSE OF EXAMPLE 3 TO A RAMP INPUT,
$t = 0$ TO 0.1 SECOND

START TIME= .0000 ACTUAL STOP TIME= .0100 PROPOSED STOP TIME= .0100

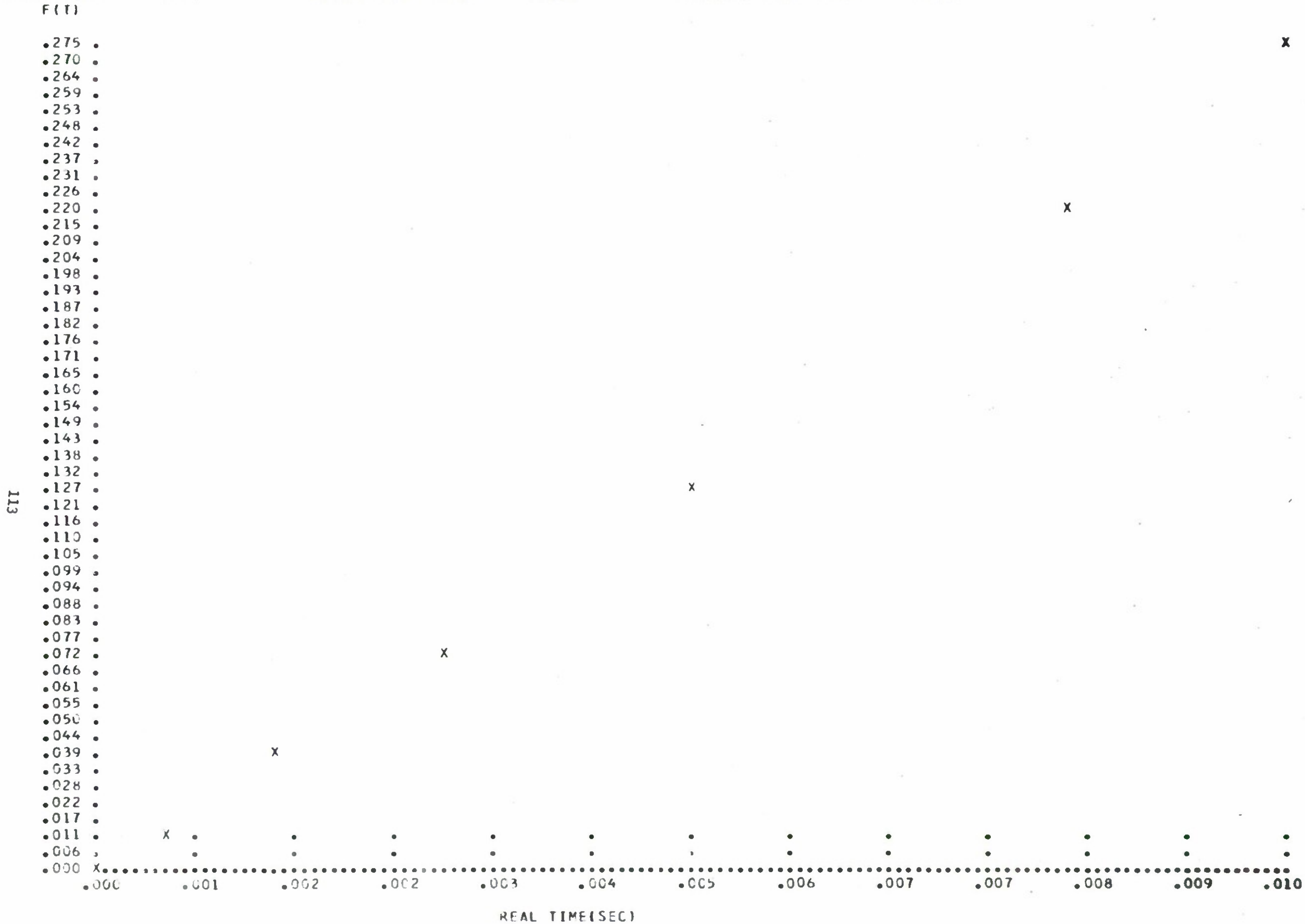

FIGURE 13. TRANSIENT RESPONSE OF EXAMPLE 3 TO A RAMP INPUT, t = 0 TO 0.01 SECOND

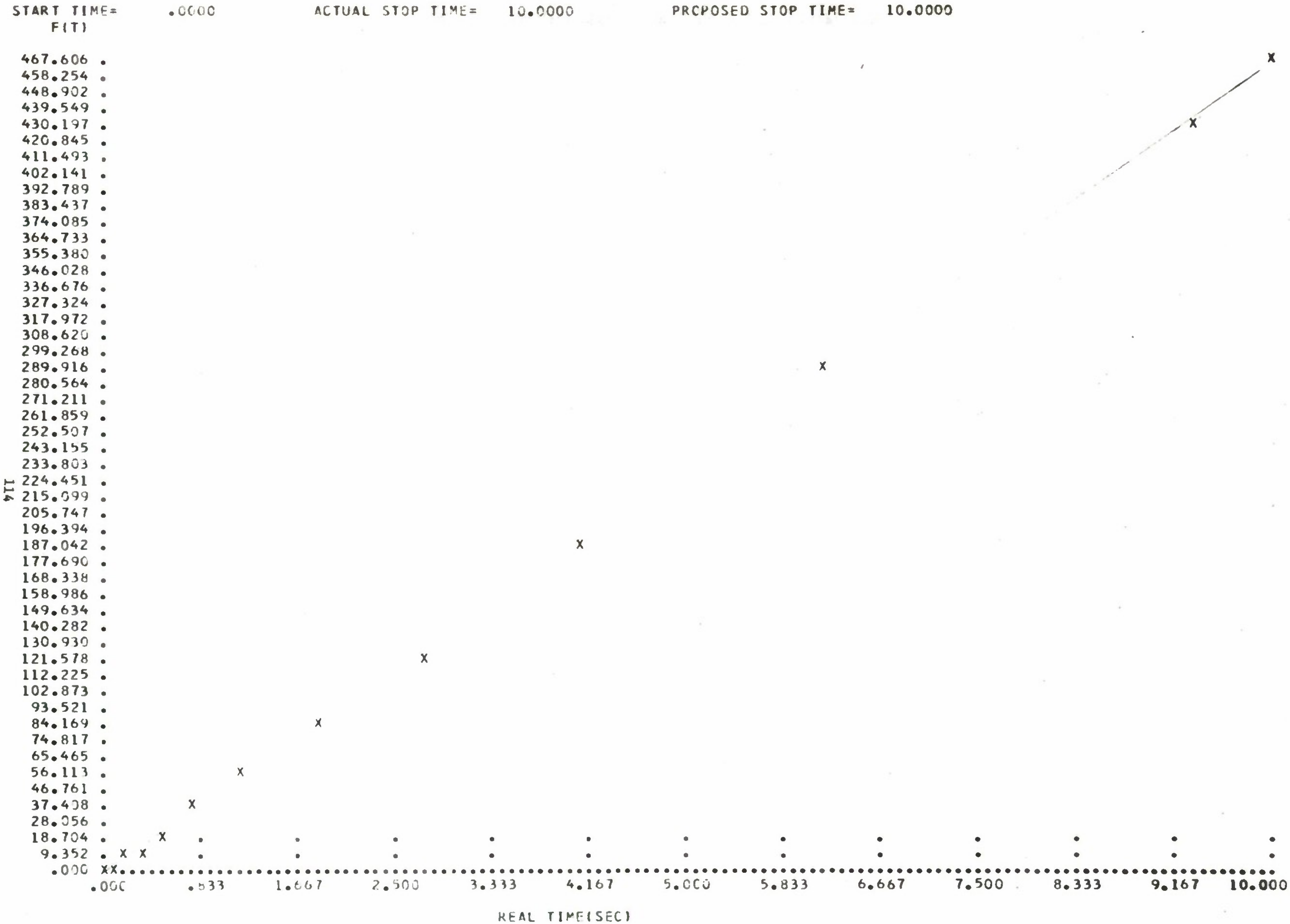

FIGURE 14. TRANSIENT RESPONSE OF EXAMPLE 3 TO A RAMP INPUT,
t = 10.0 SECONDS

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13. ABSTRACT

An existing program, which determines the inverse Laplace transform of a quotient of two polynomials, provides expanded systems analysis capability. The program complements a Root Locus Program (AMSAA Technical Memorandum No. 21) and a Frequency Response Program (AMSAA Technical Memorandum No. 69). The program uses a self-contained complex arithmetic routine and also a self-adjusting variable scale plotting technique. The plotting is done on a standard line printer and gives a time history plot of system response for a variety of input forcing functions.

A listing of the FORTRAN IV source deck and the corresponding flow chart of the program is shown in the appendixes. Also, several examples are given to introduce the user to the operating procedures and capabilities of the program.

DD FORM 1 NOV 65 1473 REPLACES DD FORM 1473, 1 JAN 64, WHICH IS OBSOLETE FOR ARMY USE.

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14. KEY WORDS	LINK A		LINK B		LINK C	
	ROLE	WT	ROLE	WT	ROLE	WT
Systems Analysis						
Systems Synthesis						
Transient Response						
Time Response						
Inverse Laplace Transform						
Time Plots						

Transfer functions
Boundary value prob.